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THE TRUNCATED EM METHOD FOR JUMP-DIFFUSION SDDES WITH SUPER-LINEARLY GROWING DIFFUSION AND JUMP COEFFICIENTS*

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Abstract

This work is concerned with the convergence and stability of the truncated Euler-Maruyama (EM) method for super-linear stochastic differential delay equations (SDDEs) with time-variable delay and Poisson jumps. By constructing appropriate truncated functions to control the super-linear growth of the original coefficients, we present two types of the truncated EM method for such jump-diffusion SDDEs with time-variable delay, which is proposed to be approximated by the value taken at the nearest grid points on the left of the delayed argument. The first type is proved to have a strong convergence order which is arbitrarily close to 1/2 in mean-square sense, under the Khasminskii-type, global monotonicity with U function and polynomial growth conditions. The second type is convergent in q-th ($q < 2$) moment under the local Lipschitz plus generalized Khasminskii-type conditions. In addition, we show that the partially truncated EM method preserves the mean-square and H_∞ stabilities of the true solutions. Lastly, we carry out some numerical experiments to support the theoretical results.

Mathematics subject classification: 60H10, 60H35, 65L20.
Key words: SDDEs, Truncated EM method, Time-variable delay, Poisson jumps.

1. Introduction

The stochastic differential delay equation (SDDE) models play a significant part in many application fields, such as economy, finance, automatic control and population dynamics (see,

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e.g., [4, 5, 10, 11, 26, 27, 29, 35, 41]). In general, such models rarely have explicit solutions available. Thus, there appears to be a practical need to estimate the true solution of the model via numerical approach. Moreover, many important SDDE models often possess super-linear growth coefficients in the real world, for example, stochastic delay Lotka-Volterra model arising in population dynamics of the form (see, e.g., [4])

$$dX(t) = \text{diag}\big(X_1(t), \cdots, X_d(t)\big)\big[(b + AX(t-\tau))dt + \sigma X(t)dB(t)\big], \tag{1.1}$$

where $B(t)$ is a Brownian motion and $\sigma = (\sigma_{ij})_{d\times d}$ is a matrix representing the intensity of noise. If we apply the explicit Euler-Maruyama (EM) scheme to the model (1.1), it is well-documented that such EM approximation fails to converge in the strong sense to the true solution of (1.1) (see, e.g., [23]).

When the delay component vanishes, the underlying SDDEs reduce to the classical stochastic differential equations (SDEs), numerical methods for which have been extensively investigated for the past decades under the global Lipschitz condition (see, e.g., [21, 25, 39]). In the setting of SDEs whose coefficients can be allowed to grow super-linearly, several explicit schemes have been introduced, including tamed EM and Runge-Kutta schemes [18, 24, 40], balanced EM schemes [46, 49] and truncated EM schemes [30, 32, 33]. Recently, the attention of some researches was attracted to the strong convergence of explicit numerical methods for super-linear SDEs with delay, i.e., SDDEs. Guo *et al.* [20] were the first to discuss the strong convergence of the truncated EM method for SDDEs under the local Lipschitz plus the generalized Khasminskii-type conditions. In a subsequent paper, Gao *et al.* [19] took the jumps into consideration and they extended convergence results from [15, 20] to the case of SDDEs with Poisson jumps. By using a different estimate for the difference between the original and the truncated coefficients, Fei *et al.* [17] relaxed the restrictive condition on the step size which is required to extremely small and thus improved the convergence results of [20]. Moreover, Song *et al.* [43] achieved a better convergence order than [17, 20] by adopting the truncation techniques from [30] for such SDDEs. Other explicit numerical methods for super-linear SDDEs, say tamed EM, balanced EM, truncated Milstein, projected EM, are discussed in [7, 12, 13, 28, 45, 48], respectively.

Numerous studies suggest strong empirical evidences that there exist jumps within financial markets (see, e.g., [1–3]). Jumps risks can not be ignored in the pricing of financial assets (see, e.g., [38]). When it comes to the convergence of numerical schemes for SDEs or SDDEs with jumps, most of the existing works impose the linear growth assumption on the jump coefficient, such as [12, 22, 39]. In the context of the super-linear growth assumption on the jump, Deng *et al.* [15] and Gao *et al.* [19] established the convergence results of the truncated EM method for jump-diffusion SDEs and SDDEs in small moment (i.e., q-th moment with q small than 2), respectively. However, when the mean-square convergence order is considered, some difficulties arise. Due to the fact that the higher moment bounds of the Poisson increments contribute to magnitude not more than $\mathcal{O}(\Delta)$, i.e.,

$$\mathbb{E}|N(t+\Delta) - N(t)|^p \le C\Delta, \quad p \ge 2, \tag{1.2}$$

where C is a positive constant independent of Δ, the order of the one-step error in the $\mathcal{L}^p$-norm drops to $1/p$ and further decreases when we apply the truncated technology (3.8) or (4.7) to super-linearly growing jump coefficients (see, e.g., [6, 8]). If the super-linear growth of the jump coefficient can not be addressed well, then the convergence rate in mean-square sense will not achieve the desired order. To overcome these difficulties, we design a truncated function depending on the Khasminskii parameter p_0 to control the super-linear growth of the jump