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UNIFORM SUPERCONVERGENCE ANALYSIS OF A TWO-GRID MIXED FINITE ELEMENT METHOD FOR THE TIME-DEPENDENT BI-WAVE PROBLEM MODELING D-WAVE SUPERCONDUCTORS*

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Abstract

In this paper, a two-grid mixed finite element method (MFEM) of implicit Backward Euler (BE) formula is presented for the fourth order time-dependent singularly perturbed Bi-wave problem for d-wave superconductors by the nonconforming EQ_1^{rot} element. In this approach, the original nonlinear system is solved on the coarse mesh through the Newton iteration method, and then the linear system is computed on the fine mesh with Taylor's expansion. Based on the high accuracy results of the chosen element, the uniform superclose and superconvergent estimates in the broken H^1- norm are derived, which are independent of the negative powers of the perturbation parameter appeared in the considered problem. Numerical results illustrate that the computing cost of the proposed two-grid method is much less than that of the conventional Galerkin MFEM without loss of accuracy.

Mathematics subject classification: 65M60, 65N12, 65N30.
Key words: Time-dependent Bi-wave problem, Two-grid mixed finite element method, Uniform superclose and superconvergent estimates.

1. Introduction

Superconductors are materials that have no resistance to the electric current at a T_c (critical temperature) [1]. In the state of low-T_c superconductivity, electrons are found to pair in a form and move together in a spherical orbit but in the opposite direction, which is often called s-wave [2] and Ginzburg-Landau-type models were generally proposed to describe this phenomenon [3]. For high-T_c superconductivity, electrons have been strongly proved to travel together in orbits as a four-leaf clover for d-wave pairing symmetry [4, 5] and researchers have studied various generalizations of Ginzburg-Landau-type models to explain high-T_c superconductors [6].

In the time-dependent versions of Ginzburg-Landau-type for d-wave superconductor [7-9], there exist two scalar order parameters ψ_s and ψ_d whose magnitudes represent the density of superconducting charge carriers and the parameter $\delta = -1/\beta$, where β is related to the ratio $ln(T_{s0}/T)/ln(T_{d0}/T)$ with T_{s0} and T_{d0} being the critical temperatures of s-wave and d-wave components. Some studies have shown that when $\beta \to -\infty$, s-wave component diminished and d-wave component became the leading term [8], and superconductor will be completely d-wave as $T \to T_{d0}$ ($T_{s0} < T_{d0}$) [9]. Therefore, the following fourth order time-dependent singularly

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perturbed Bi-wave problem emerged from the time-dependent Ginzburg-Landau-type model in the case $\beta \to -\infty$:

$$\begin{cases} \psi_t + \delta\theta^2\psi - \Delta\psi + f(\psi) = 0, & (X,t) \in \Omega \times J, \\ \psi = \dfrac{\partial\psi}{\partial\bar{n}} = 0, & (X,t) \in \partial\Omega \times J, \\ \psi(X,0) = \psi_0(X), & X \in \Omega, \end{cases} \tag{1.1}$$

where $X = (x,y)$, $J = (0,T]$, $\psi_t = \partial\psi/\partial t$, θ is the bi-wave operator,

$$\theta\psi = \frac{\partial^2\psi}{\partial x^2} - \frac{\partial^2\psi}{\partial y^2}, \quad \theta^2\psi = \frac{\partial^4\psi}{\partial x^4} - 2\frac{\partial^4\psi}{\partial x^2 y^2} + \frac{\partial^4\psi}{\partial y^4}, \quad \bar{n} = (n_1, -n_2), \quad \frac{\partial\psi}{\partial\bar{n}} = \nabla\psi\cdot\bar{n},$$

$\Omega \subset R^2$ is a bounded domain with the boundary $\partial\Omega$, and $n = (n_1, n_2)$ denotes the unit outward normal to $\partial\Omega$, $f(\psi) = \psi^3 - \psi$ and $\psi_0(X)$ is a known smooth function. Hence, $0 < \delta < 1$ is expected to be small for d-wave superconductors and problem (1.1) degenerates into the semilinear parabolic equation when $\delta \to 0$.

In recent years, there are some theoretical analysis and numerical simulations about FEMs, such as optimal order error estimates of conforming Galerkin FEMs and the modified Morley-type discontinuous Galerkin FEMs in [10, 11], uniform superconvergence error estimates of Ciarlet-Raviart schemes with the conforming and nonconforming elements in [12–14]. But these works mainly focused on the stationary singularly perturbed Bi-wave problems. Thus, to develop an effective computational method for investigating problem (1.1) is of more practical significance. As a highly efficient and accurate method, the two-grid method was proposed by [15, 16] for the nonsymmetric and nonlinear problems and has been well applied to deal with many types of problems for optimal or superconvergent error estimates, such as parabolic equation [17, 18], hyperbolic problem [19], Ginzburg-Landau equation [20], Benjamin-Bona-Mahony equation [21], and so on. Nevertheless, no studies on uniform superconvergence analysis of two-gird MFEM for problem (1.1) exists in the literature and whether the error estimate result will be relevant to the negative powers of the perturbation parameter δ or not still remains open.

In this paper, as a first attempt, we will formulate a two-grid efficient algorithm of MFEM for problem (1.1) by the nonconforming EQ_1^{rot} element and analyze the corresponding uniform superclose and superconvergence behavior, which is independent of the negative powers of the parameter. The main reasons for the uniform error estimates are as follows: the first is that the suited approximation scheme is developed, the second is that the special characters of the chosen element are employed (see the formulas (2.1)-(2.3) below), the third is that the equation includes the positive term $-\Delta$. The reminder of the paper is organized as follows. In Section 2, the stability of the numerical solution is proved and uniform superconvergence result of the semi-discrete scheme for problem (1.1) is derived. In Section 3, the superclose estimates of order $O(H^2 + \tau)$ and order $O(H^4 + h^2 + \tau)$ for the two-grid method are demonstrated, respectively, where H and h are the subdivision parameters on the coarse and fine meshes, and τ the time step. Moreover, the corresponding global superconvergence result of order $O(H^4 + h^2 + \tau)$ is obtained through the interpolated postprocessing approach. It should be mentioned that the error estimates obtained herein is independent of the negative powers of the perturbation parameter δ by use of the high accuracy characters of the selected element. In the last section, some numerical results are conducted to confirm the theoretical analysis and indicate that the computing cost of the proposed two-grid method is less than a half of the traditional Galerkin MFEM.