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Novel Partitioned Time-Stepping Algorithms for Fast Computation of Random Interface-Coupled Problems with Uncertain Parameters

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Abstract. The simulation of multi-domain, multi-physics mathematical models with uncertain parameters can be quite demanding in terms of algorithm design and computation costs. Our main objective in this paper is to examine a physical interface coupling between two random dissipative systems with uncertain parameters. Due to the complexity and uncertainty inherent in such interface-coupled problems, uncertain diffusion coefficients or friction parameters often arise, leading to considering random systems. We employ Monte Carlo methods to produce independent and identically distributed deterministic heat-heat model samples to address random systems, and adroitly integrate the ensemble idea to facilitate the fast calculation of these samples. To achieve unconditional stability, we introduce the scalar auxiliary variable (SAV) method to overcome the time constraints of the ensemble implicit-explicit algorithm. Furthermore, for a more accurate and stable scheme, the ensemble data-passing algorithm is raised, which is unconditionally stable and convergent without any auxiliary variables. These algorithms employ the same coefficient matrix for multiple linear systems and enable easy parallelization, which can significantly reduce the computational cost. Finally, numerical experiments are conducted to support the theoretical results and showcase the unique features of the proposed algorithms.

AMS subject classifications: 65M55, 65M60

Key words: Scalar auxiliary variable, ensemble algorithm, random interface-coupled problems, implicit-explicit partitioned method, data-passing partitioned method.

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1. Introduction

In the realm of mathematical physics, models of atmosphere-ocean interaction are commonly constructed by two incompressible Newtonian fluids along with compatible interface conditions [2, 20–22]. Recently, much focus has been placed on multi-domain, multi-physics coupled problems [1, 3, 8, 10, 16, 28], with particular interest in novel numerical simulations for fluid-fluid interaction models [5–7, 29]. In the study outlined by [6], a simplified atmosphere-ocean interaction model is considered, featuring a deterministic friction parameter κ and is deemed a linear heat-heat coupled system. To decouple such a multi-domain, multi-physics system naturally resulting in parallel computation, Connors *et al.* [6] presented the implicit-explicit (IMEX) and data-passing partitioned methods, which are both first-order in time, fully discrete methods. The most noteworthy aspect of [6] is illustrated by the fact that the data-passing partitioned method is unconditionally stable and convergent. Subsequently, in the case of the atmosphere-ocean interaction model incorporating some nonlinear interface demands, Connors *et al.* [7] built on existing research [6] and furthered an unconditionally stable method by executing geometric averaging for nonlinear terms.

Due to the inaccuracy of observation data, the complexity of the atmosphere-ocean coupling, or the introduction of additional uncertainty sources, the friction parameter κ [5] and diffusion coefficients ν_1, ν_2 [23] are physically impossible to determine, which can only give an approximate value range or meet a certain probability distribution. Therefore, exploring the numerical simulation of such problems with uncertain inputs is necessary. Uncertain parameters are often considered random functions determined by specific covariance structures, typically experimentally constructed basic random fields. In this paper, we focus on the linear heat-heat interface-coupled problems with three random coefficients κ, ν_1, and ν_2 as simplified fluid-fluid models. The main issue of this paper is to establish effective numerical schemes based on the ensemble idea, existing IMEX, and data-passing partitioned schemes, to achieve unconditional stability and fast computation for the random interface-coupled model.

One of the most popular approaches to address random problems is the Monte Carlo method [23, 24]. This involves transforming these random problems into a series of traditional PDEs that can be tackled by existing standard numerical methods. However, the solution's uncertainty and sensitivity require more samples for better estimation, which can lead to slow convergence rates. As a result, numerous linear equations with different stiffness matrices are formed, necessitating extensive computational costs. To increase efficiency and tackle these computational challenges, researchers proposed a fast ensemble time-stepping algorithm [12] to solve J (the number of samples) Navier-Stokes equations with different initial conditions and forcing terms. Jiang *et al.* [12] skillfully put forward the ensemble idea, at each time step, to solve the linear systems with a shared coefficient matrix and J right-hand sides. As only one efficient iterative solver was required for multiple linear systems with a common coefficient matrix, this significantly reduced storage requirements and computing costs. For uncertainties in initial conditions and forcing terms, the ensemble algorithm