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ASYMPTOTIC THEORY FOR THE CIRCUIT ENVELOPE ANALYSIS*

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Abstract

Asymptotic theory for the circuit envelope analysis is developed in this paper. A typical feature of circuit envelope analysis is the existence of two significantly distinct timescales: one is the fast timescale of carrier wave, and the other is the slow timescale of modulation signal. We first perform pro forma asymptotic analysis for both the driven and autonomous systems. Then resorting to the Floquet theory of periodic operators, we make a rigorous justification for first-order asymptotic approximations. It turns out that these asymptotic results are valid at least on the slow timescale. To speed up the computation of asymptotic approximations, we propose a periodization technique, which renders the possibility of utilizing the NUFFT algorithm. Numerical experiments are presented, and the results validate the theoretical findings.

Mathematics subject classification: 94C05, 35C20, 35B25, 35B40, 34C25.
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1. Introduction

Radio frequency (RF) simulation presents a major challenge in the design process of wireless products. The traditional transient analysis is time-consuming for RF systems due to the strong nonlinearity and the multirate characteristic. The well-known harmonic balance method [11] is efficient and accurate for computing the periodic steady-state (PSS) response of nonlinear circuits. However, harmonic balance becomes inappropriate when the signal cannot be represented by the sum of relatively few discrete sinusoids or tones.

Circuit envelope analysis allows circuits with complicated modulated RF signals to be simulated much more efficiently. For a general RF circuit governed by the following equation:

$$f\big(\dot{x}(t), x(t), t, b(t)\big) = 0, \quad \forall t > 0,$$

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the circuit envelope analysis assumes that the modulation signal $b(t)$ varies slowly with respect to the carrier, and aims to predict long time behavior of the circuit, rather than the transient response [9]. The envelope methods proposed in [2,6,13,16], often referred to as Fourier envelope methods, are generalizations of harmonic balance method. In these methods, the signals are assumed to take the form of Fourier expansion with slowly time-varying coefficients. Methods of this type could possibly fail without appropriate choice of initial condition [18]. Another type of envelope methods, termed as envelope following methods, are based on transient analysis, see [1, 10, 15]. The main idea of such methods is to jump over some carrier periods, and thus diminish the number of time steps. However, some benchmark circuits show that the existing implementations of envelope following are not competitive with transient analysis, due to the repeated trials made to find the optimal time jump [3]. Furthermore, the multi-time PDE (MPDE) technique is also an alternative choice for circuit envelope analysis [12, 14, 17]. In the previous works on circuit envelope analysis, rigorous mathematical analysis is seldom involved, which makes it difficult to explain the causes when undesirable phenomena occur.

In this paper, we develop asymptotic theory for the circuit envelope analysis. To the best of authors' knowledge, investigations of this kind have not been conducted yet. Using two time analysis, we transform the governing ODEs of RF circuits into singularly perturbed PDEs with well-prepared initial values. We then perform a perturbation analysis to derive an asymptotic solution. With the help of Floquet theory, we give rigorous error estimates.

A novel envelope analysis method is proposed based on the developed asymptotic theory. In this method, one needs to first solve a parameterized PSS system, and then derive an asymptotic solution by a fast interpolation scheme. The proposed method works for both driven (i.e., non-autonomous) and autonomous circuits. Particularly, in the autonomous case, the perturbation projection vector (PPV) [4] needs to be computed to fix a phase shift term. The error estimate validates the effectiveness of the proposed method, as the asymptotic parameter ε, i.e., the ratio of the frequency of modulation signal and the frequency of the carrier, is relatively small.

The rest of this paper is organized as follows. In Section 2, we review the Floquet theory of periodically time-varying systems of linear ODEs and work out stability estimates for the singularly perturbed Cauchy problem. In Sections 3 and 4, we develop asymptotic theory for the driven and autonomous circuit envelope analyses, respectively. In Section 5, we perform numerical experiments on some model RF circuits of either type. We conclude this paper in Section 6.

2. Floquet Theory

Let us consider linear operators of the following form:

$$(\mathcal{L}x)(\tau) \stackrel{def}{=} M_1(\tau)\frac{d}{d\tau}[M_2(\tau)x(\tau)] + A(\tau)x(\tau), \tag{2.1}$$

where $x(\cdot):\mathbb{R}\to\mathbb{R}^d$ is a smooth function, and $M_1(\tau), M_2(\tau)$ and $A(\tau)$ are all smooth T-periodic functions valued in $\mathbb{R}^{d\times d}$. We assume that the matrix $M_1(\tau)M_2(\tau)$ is nonsingular for all $\tau\in\mathbb{R}$. It is known that the following initial value problem:

$$M_1(\tau)\frac{d}{d\tau}[M_2(\tau)x(\tau)] + A(\tau)x(\tau) = 0, \quad x(s) = x_s$$

admits a unique smooth solution $x(\tau)$ for any $s\in\mathbb{R}$ and any $x_s\in\mathbb{R}^d$. Therefore, the transition state matrix, denoted by $X(\tau;s)$ which maps the initial data x_s to $x(\tau)$, satisfies the following