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Simultaneous Approximation for Szász-Mirakian-Stancu-Durrmeyer Operators

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Abstract. The aim of this work is to generalize Szász-Mirakian operator in the sense of Stancu-Durrmeyer operators. We obtain approximation properties of these operators. Here we study asymptotic as well as rate of convergence results in simultaneous approximation for these modified operators.

Key Words: Szász-Mirakian-Stancu-Durrmeyer operator, simultaneous approximation, asymptotic, rate of convergence.

AMS Subject Classifications: 41A20, 41A25, 41A35

1 Introduction

Let α and β be two non-negative parameters satisfying the condition $0\le\alpha\le\beta$. For any nonnegative integer n,

$$f\in C[0,\infty)\to S_n^{(\alpha,\beta)}f,$$

the Stancu type Szász-Mirakian-Durrmeyer operators are defined by

$$S_{n,r}^{(\alpha,\beta)}(f,x)=n\sum_{k=0}^{\infty}s_{n,k}(x)\int_0^{\infty}s_{n,k+r}(t)f\Big(\frac{nt+\alpha}{n+\beta}\Big)dt, \tag{1.1}$$

where

$$s_{n,k}(x)=e^{-nx}\frac{(nx)^k}{k!}.$$

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For $\alpha=\beta=0$ these operators become the well known Szász-Mirakian-Durrmeyer operators

$$S_n^{(0,0)}(f,x)=S_n(f,x)$$

introduced by Mazhar and Totik [3]. In [1] the author established some direct results in simultaneous approximation for this special case. Gupta et al. [2] estimated the rate of convergence for functions having derivatives of bounded variation for this special case $\alpha=\beta=r=0$. Also for this special case [4] estimated the rate of convergence for the Bézier variant of Szász-Mirakian-Durrmeyer operators.

The purpose of this paper is to study approximation properties of the Stancu type Szász-Mirakian-Durrmeyer operators. We give the rate of convergence and Voronovskaya type asymptotic result for the same operators.

2 Basic results

In this section we establish a recurrence formula for the moments.

For simultaneous approximation, we need the following form of the operators (1.1)

$$S_{n,r}^{(\alpha,\beta)}(f,x)=n\sum_{k=0}^{\infty}s_{n,k}(x)\int_0^{\infty}s_{n,k+r}(t)f\Big(\frac{nt+\alpha}{n+\beta}\Big)dt.$$

Lemma 2.1. *For $n,m\in\mathbf{N}\cup\{0\}$, $0\le\alpha\le\beta$, let us consider*

$$\mu_{n,m,r}^{(\alpha,\beta)}(x)=S_{n,r}^{(\alpha,\beta)}\big((t-x)^m,x\big)=n\sum_{k=0}^{\infty}s_{n,k}(x)\int_0^{\infty}s_{n,k+r}(t)\Big(\frac{nt+\alpha}{n+\beta}-x\Big)^m dt,$$

we get

$$\mu_{n,0,r}^{(\alpha,\beta)}(x)=1,\qquad \mu_{n,1,r}^{(\alpha,\beta)}(x)=\frac{\alpha+r+1-\beta x}{n+\beta},$$
$$\mu_{n,2,r}^{(\alpha,\beta)}(x)=\frac{\beta^2x^2+2(n-\alpha\beta-\beta-\beta r)x+(\alpha+r+1)(\alpha+r+2)-\alpha}{(n+\beta)^2},$$

and

$$(n+\beta)\mu_{n,m+1,r}^{(\alpha,\beta)}(x)=x[\mu_{n,m,r}^{(\alpha,\beta)}(x)]'+(m+\alpha+r+1-\beta x)\mu_{n,m,r}^{(\alpha,\beta)}(x)+m\Big(\frac{2(n+\beta)x-\alpha}{n+\beta}\Big)\mu_{n,m-1,r}^{(\alpha,\beta)}(x). \tag{2.1}$$

Proof. By simple calculation we can easily obtain

$$xs'_{n,k}(x)=(k-nx)s_{n,k}(x).$$