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Remarks on Exponential Stability of Solutions for the Compressible p-th Power Newtonian Fluid with Large Initial Data

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Abstract. In this paper, we establish the exponential stability of the global spherically and cylindrically symmetric solutions in H^i $(i=1,2,4)$ for the p-th power Newtonian fluid in multi-dimension with large initial data. The key point is that the smallness of initial data is not needed if the initial data are cylindrically symmetric.

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1 Introduction

In this paper, we establish the exponential stability of global solutions in H^i $(i=1,2,4)$ for the compressible Navier-Stokes equations which describe the motion of the p-th power Newtonian fluid. We assume that pressure P, in terms of the density ρ and absolute temperature θ, is given by $P=R\rho^p\theta$ with constant $R>0$ and the pressure exponent $p\geq 1$, and the specific internal energy $e=c_v\theta$ with constant specific heat coefficient $c_v>0$. We assume that the corresponding solutions only depend on the radial variable r and the time variable $t\in[0,+\infty)$. Here we use $\overrightarrow{\mathbf{U}}(\mathbf{x},t)$ to denote the velocity of fluid and $\Omega=$

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$\{\mathbf{x}|0<a\le|\mathbf{x}|\le b<+\infty\}$ is a symmetric domain in $\mathbb{R}^N$. As in [1–3], the equations can be read as

$$\begin{cases}\rho_t+(\rho u)_r+\dfrac{m\rho u}{r}=0,\\ \rho\left(u_t+uu_r-\dfrac{v^2}{r}\right)+P_r=\beta\left(u_r+\dfrac{mu}{r}\right)_r,\\ \rho\left(v_t+uv_r+\dfrac{uv}{r}\right)=\mu\left(v_r+\dfrac{mv}{r}\right)_r,\\ \rho(w_t+uw_r)=\mu\left(w_{rr}+\dfrac{mw_r}{r}\right),\\ c_v\rho(\theta_t+u\theta_r)+P\left(u_r+\dfrac{mu}{r}\right)=k\left(\theta_{rr}+\dfrac{m\theta_r}{r}\right)+Q,\end{cases} \tag{1.1}$$

where the viscosity coefficients μ,λ satisfy the natural restrictions $\mu>0$, $N\lambda+2\mu\ge0$ and the coefficient of heat conduction k is positive $k>0$, $\beta=2\mu+\lambda$ and

$$Q=\lambda\left(u_r+\frac{mu}{r}\right)^2+\mu\left\{\left(v_r-\frac{mv}{r}\right)^2+w_r^2+2u_r^2+\frac{2mu^2}{r^2}\right\}.$$

In the spherically symmetric case, $m=N-1, r=|\mathbf{x}|, \overrightarrow{\mathbf{U}}(\mathbf{x},t)=u(r,t)\frac{\mathbf{x}}{r}, v=w=0$. In the cylindrically symmetric case, $m=1, r=\sqrt{x_1^2+x_2^2}$, and

$$\overrightarrow{\mathbf{U}}(\mathbf{x},t)=u(r,t)\frac{(x_1,x_2,0)}{r}+v(r,t)\frac{(-x_2,x_1,0)}{r}+w(r,t)(0,0,1).$$

The following boundary and initial conditions can be given by

$$(u,v,w,\theta_r)(a,t)=(u,v,w,\theta_r)(b,t)=0,\qquad t\in[0,+\infty), \tag{1.2}$$

$$(\rho,u,v,w,\theta)(r,0)=(\rho_0,u_0,v_0,w_0,\theta_0)(r),\qquad r\in[a,b]. \tag{1.3}$$

As in [1,4,5], we denote by $\eta=\frac{1}{\rho}$ the specific volume of the flows and can transfer the problem (1.1)-(1.3) into the equations in Lagrangian coordinates as follows

$$\begin{cases}\eta_t=(r^mu)_x,\qquad 0<x<L,\ t>0, & (1.4)\\ u_t=r^m\left(-\dfrac{R\theta}{\eta^p}+\beta\dfrac{(r^mu)_x}{\eta}\right)_x+\dfrac{v^2}{r}, & (1.5)\\ v_t=\mu r^m\left(\dfrac{(r^mv)_x}{\eta}\right)_x-\dfrac{uv}{r}, & (1.6)\\ w_t=\mu r^m\left(\dfrac{(r^mw)_x}{\eta}\right)_x+\dfrac{\mu m\eta w}{r^2}, & (1.7)\\ c_v\theta_t=\left(-\dfrac{R\theta}{\eta^p}+\beta\dfrac{(r^mu)_x}{\eta}\right)(r^mu)_x+\left(k\dfrac{r^{2m}\theta_x}{\eta}\right)_x+\mu\dfrac{(r^mv)_x^2}{\eta}\\ \qquad\qquad+\mu\dfrac{r^{2m}w_x^2}{\eta}-2\mu m(r^{m-1}u^2+r^{m-1}v^2)_x. & (1.8)\end{cases}$$