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Existence and Uniqueness of Strong Solution for Shear Thickening Fluids of Second Grade

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Abstract. In this paper we study the equations governing the unsteady motion of an incompressible homogeneous generalized second grade fluid subject to periodic boundary conditions. We establish the existence of global-in-time strong solutions for shear thickening flows in the two and three dimensional case. We also prove uniqueness of such solution without any smallness condition on the initial data or restriction on the material moduli.

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1 Introduction

The theory and the applications of non-Newtonian fluids have attracted the attention of many scientists for a long time since they are more appropriate than Newtonian fluids in many areas of engineering sciences such as geophysics, glaciology, colloid mechanics, polymer mechanics, blood and food rheology, etc. Fluids with shear dependent viscosity, which can exhibit shear thinning and shear thickening and include the power-law fluids as a special case, constitute a large class of non-Newtonian fluids. For instance, see [1–3] for more detailed discussions.

There are many models of non-Newtonian fluids which have recently become to be of great interest. Among these models one can cite fluids of differential type. The second

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grade fluids, which are a subclass of them, have been successfully investigated in various kinds of flows of materials such as oils, greases, blood, polymers, suspensions, and liquid crystals. In the classical incompressible fluids of second grade, it is customary to assume that the Cauchy stress tensor $\mathbf{T}$ is related to the velocity gradient ∇u and its symmetric part $\mathbf{D}u$ through the relation

$$\mathbf{T}=-p\mathbf{I}+\mu\mathbf{A}_1+\alpha_1\mathbf{A}_2+\alpha_2\mathbf{A}_1^2, \tag{1.1}$$

where p is the indeterminate part of the stress due to the constraint of incompressibility, μ is the kinematic viscosity, and α_1 and α_2 are material moduli which are usually referred to as the normal stress coefficients. The kinematical tensor $\mathbf{A}_1$ and $\mathbf{A}_2$ are the first and the second Rivlin-Ericksen tensor, respectively, and they are defined through

$$\mathbf{A}_1=\nabla u+\nabla u^t=2\mathbf{D}u=\nabla u+(\nabla u)^t, \tag{1.2}$$

$$\mathbf{A}_2=\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{A}_1+\mathbf{A}_1\nabla u+\nabla u^t\mathbf{A}_1, \tag{1.3}$$

where $u=(u_1,\cdots,u_d)$ and $\nabla u:=(\partial_j u_i)_{1\le i,j\le d}$ ($d=2$ or 3 is the space dimension) denote the velocity vector field and its gradient. Here $\partial_j u_i$ stands for the partial space derivative of u_i with respect to x_j, $i,j=1,\cdots,d$. The material derivative is given by

$$\frac{\mathrm{d}}{\mathrm{d}t}(\cdot)=\partial_t(\cdot)+(u\cdot\nabla)(\cdot), \tag{1.4}$$

where ∂_t is the partial derivative with respect to time and $(u\cdot\nabla)$ the differential operator with respect to the spatial variable defined by $u\cdot\nabla=\sum_{i=1}^d u_i\partial_i$.

Indeed, in [4], Dunn studied the thermodynamics and stability of second-grade fluids with viscosity μ depending on the shear rate $|\mathbf{D}u|$ (i.e. the Euclidean norm of the symmetric part of the velocity gradient defined by $|\mathbf{D}u|=\sqrt{\mathrm{Tr}(\mathbf{D}u)^2}$) and showed that if the fluid is to be compatible with thermodynamics in the sense that all motions of the fluid meet the Clausius-Duhem inequality and the assumption that the specific Helmholtz free energy of the fluid be minimum in equilibrium, then μ,α_1 and α_2 in (1.1) must verify

$$\sqrt{\frac{3}{2}}\frac{\mu(|\mathbf{D}u|)}{|\mathbf{D}u|}\le\alpha_1+\alpha_2\le\sqrt{\frac{3}{2}}\frac{\mu(|\mathbf{D}u|)}{|\mathbf{D}u|}. \tag{1.5}$$

In the 1980s, Man, Kjartanson and coworkers [5] and [6] showed that polycrystalline ice in creeping flow under pressuremeter tests can be modeled as an incompressible second-grade fluid with the viscosity depending on the shear rate. The constitutive equation proposed by Man and coworkers, which leads to well-posed initial-boundary-value problems in nonsteady channel flow [7] and can also model the flow of ice in triaxial creep tests [8], is

$$\mathbf{T}=-p\mathbf{I}+\mu(|\mathbf{D}u|)\mathbf{A}_1+\alpha_1\mathbf{A}_2+\alpha_2\mathbf{A}_1^2, \tag{1.6}$$