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Global Solvability in Thermoelasticity with Second Sound on the Semi-Axis

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Abstract. In this paper, we consider initial boundary value problem for the equations of one-dimensional nonlinear thermoelasticity with second sound in $\mathbb{R}^+$. First, we derive decay rates for linear systems which, in fact, is a hyperbolic systems with a damping term. Then, using this linear decay rates, we get L^1 and L^∞ decay rates for nonlinear systems. Finally, combining with L^2 estimates and a local existence theorem, we prove a global existence and uniqueness theorem for small smooth data.

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1 Introduction

This paper is concerned with the equations for the one-dimensional nonlinear thermoelasticity with second sound with reference configuration in $\mathbb{R}^+$, which can be described as follows: (see [1] and [2])

$$\omega_t - v_x = 0, \tag{1.1}$$
$$v_t - a(\omega,\theta,q)\omega_x + b(\omega,\theta,q)\theta_x = 0, \tag{1.2}$$
$$\tilde{a}(\omega,\theta,q)\theta_t + b(\omega,\theta,q)v_x + c(\theta)q_x = 0, \tag{1.3}$$
$$\tau q_t + q + \kappa\theta_x = 0, \tag{1.4}$$

where $x \in \Omega = (0,+\infty)$, $t \in (0,+\infty)$. $\omega = \omega(x,t)$, $v = v(x,t)$, $\theta = \theta(x,t)$, $q = q(x,t)$ stand for the displacement gradient, the velocity, the difference of temperature and the heat flux, respectively, and

$$a(\omega,\theta,q) = \psi_{\omega\omega}, \quad b = -\psi_{\omega,\theta}, \quad \tilde{a}(\omega,\theta,q) = -\psi_{\theta\theta}, \quad c(\theta) = 1/(\theta + T_0),$$

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where $\psi=\psi(\omega,\theta,q)$ is the specific Helmholtz free energy, τ, κ are positive constants. $|\theta|\leq K<T_0$ will be a posterior estimate justified by the global small solution. Subscripts denote partial differentiations. We consider traction free and constant temperature for all time on the boundary $\partial\Omega$, i.e.

$$\omega|_{\partial\Omega}=\theta|_{\partial\Omega}=0, \qquad t\geq 0, \tag{1.5}$$

and initial conditions

$$\omega(0,x)=\omega_0, \quad v(0,x)=v_0, \quad \theta(0,x)=\theta_0, \quad q(0,x)=q_0, \qquad x\in\Omega. \tag{1.6}$$

The above system models the second sound phenomenon. Specifically, Eq. (1.4) represents Cattaneo's Law of heat conduction modeling thermal disturbances as wave-like pulses travel at finite speed. For a discussion of this model, see [3–5]. When $\tau=0$, Eq. (1.4) turns into

$$q+\kappa\theta_x=0. \tag{1.7}$$

Eqs. (1.1)-(1.3) and (1.7) constitute the classical thermoelasticity where thermal behavior is described by the Fourier's Law, i.e., (1.7). For the comparison of the two models, see [6–9].

In the one-dimension case, the Cauchy problem for the above mentioned system has been treated by Tarabek [10], where he showed well-posedness and decay to an equilibrium. For initial boundary value problems, Racke [6] has proved the exponential stability and global existence on bounded domain. In our case, we consider a special unbounded domain, that is, $\Omega=\mathbb{R}^+$. The main difficulty here is that we can not use Poincaré's inequality since the domain is unbounded.

This paper is mainly motivated by Jiang's paper [11]. In that paper, he was able to prove a global solution for the equations of classical one-dimensional thermoelasticity in $\mathbb{R}^+$ for small smooth data. It seems that many results in classical thermoelasticity can be extended to thermoelasticity with second sound, see [6, 8, 9]. However, it is not true, for example, for Timoshenko-type thermoelastic systems, where a system can be or remain exponentially stable under Fourier's law, while it loses this property under Cattaneo's law, see [7]. Our question is that whether a weak damping effect given by Eq. (1.4) is still predominating to ensure decay rates and global solution compared with a strong impact of dissipation induced by Eq. (1.7).

We now introduce some notations which will be frequently used throughout the paper. For a non-negative integer N, let

$$D^N u=\sum_{l+m=N}\partial_t^l\partial_x^m u.$$

We denote by $W^{m,p}(\Omega)$, $0\leq m\leq\infty$, $1\leq p\leq\infty$, the usual Sobolev space with the norm $\|\cdot\|_{W^{m,p}}$. For convenience, $H^m(\Omega)$ and $L^p(\Omega)$ stand for $W^{m,2}(\Omega)$ and $W^{0,p}(\Omega)$ respectively. Let X be a Banach space. We denote by $L^p([\alpha,\beta],X)\,(1\leq p\leq\infty)$ and $\|\cdot\|_{L^p([\alpha,\beta],X)}$ the space