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Nonlinear Hyperbolic-Parabolic System Modeling Some Biological Phenomena

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Abstract. In this paper, we study a nonlinear hyperbolic-parabolic system modeling some biological phenomena. By semigroup theory and Leray-Schauder fixed point argument, the local existence and uniqueness of the weak solutions for this system are proved. For the spatial dimension $N=1$, the global existence of the weak solution will be established by the bootstrap argument.

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1 Introduction

The movement behavior of most species is guided by external signals, such as, amoeba moving upwards chemical gradients, insects orienting towards light sources. Let $u(x,t)$ and $v(x,t)$ represent the population of an organism and an external signal at place $x \in \Omega \subset R^N$ and time t respectively. It is well known that the external signal is produced by the individuals, which is described by a nonlinear function $g(v,u)$. If the spatial spread of the external signal is driven by diffusion, the full system for u and v reads (see [1-3])

$$u_t = \nabla(d\nabla u - \chi(v)\nabla v \cdot u), \tag{1.1}$$

$$v_t = d\Delta v + g(v,u). \tag{1.2}$$

Depending on what the type of the external stimulus is, one distinguishes among chemotaxis, haptotaxis, aerotaxis, geotaxis and others. Taking in account of that the external stimulus were based on the light (or the electromagnetic wave), Chen and Wu [4]

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introduced a hyperbolic-parabolic-type chemotaxis system as follows:

$$u_t=\nabla(d\nabla u-\chi(v)\nabla v\cdot u), \tag{1.3}$$
$$v_{tt}=d\Delta v+g(v,u). \tag{1.4}$$

In [4], Chen and Wu considered the systems (1.3)–(1.4) with

$$g(v,u)=-v+f(u)$$

on a bounded open domain Ω with smooth boundary. For the Neumann boundary problem, they showed the local existence and uniqueness of the solutions, and also achieved the global existence and uniqueness of the solutions of systems (1.3)–(1.4) for $N=1$. In this paper, taking our attention to the case that $g(v,u)$ is nonlinear, on a N-D, compact Riemanian manifold M without boundary, we will obtain some results similar to those given in [4].

Throughout this article, we assume that

$$1<\sigma<2, \tag{1.5}$$
$$N<2\sigma<N+2, \tag{1.6}$$
$$\frac{N}{\sigma-1}<p<\frac{2N}{N-2(\sigma-1)}, \tag{1.7}$$
$$p\geq 4, \tag{1.8}$$

where σ,p are some constants.

It is easy to check that there exist some constants σ and p such that the above four conditions can be satisfied simultaneously for $1\leq N\leq 3$. In fact, we take $\sigma=11/8$ for $N=1$, $\sigma=13/8$ for $N=2$ and $\sigma=15/8$ for $N=3$, which satisfy (1.5) and (1.6), at the same time there exists some constant p such that (1.7) and (1.8) are stratified. The conditions of (1.5)-(1.8) are crucial to our proof of the main results, since the conditions ensure the relevant Sobolev theorems. Set

$$\begin{aligned}
&X_{t_0}=C([0,t_0],H^{\sigma}(M)), \quad X_{\infty}=C([0,+\infty),H^{\sigma}(M)),\\
&Y_{t_0}=C([0,t_0],H^2(M))\cap C^1([0,t_0],H^1(M)),\\
&Y_{\infty}=C([0,+\infty),H^2(M))\cap C^1([0,+\infty),H^1(M)),\\
&Z_{t_0}=C^1([0,t_0],L^2(M)), \quad Z_{\infty}=C^1([0,\infty),L^2(M))),\\
&W_{t_0}=C^2([0,t_0],L^2(M)), \quad W_{\infty}=C^2([0,\infty),L^2(M)),
\end{aligned}$$

where M is a N-D, compact Riemannian manifold without boundary.