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THE EXISTENCES OF POSITIVE SOLUTIONS FOR P-MEAN CURVATURE OPERATOR WITH SUPERCRITICAL POTENTIAL*

Fu Hongzhuo
(School of Mathematical Siences, South China University of Technology, Guangzhou, 510640;
Department of Mathematics, University of Science and Technology of China,
Hefei, 230026, China)
(E-mail: hzhfu@scut.edu.cn)
Shen Yaotian
(School of Mathematical Siences, South China University of Technology,
Guangzhou, 510640, China)
(E-mail: maytshen@scut.edu.cn)
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Abstract This paper is concerned with the existences of positive solutions of the following Dirichlet problem for p-mean curvature operator with supercritical potential:

$$\begin{cases} -\mathrm{div}((1+|\nabla u|^2)^{\frac{p-2}{2}}\nabla u) = \lambda u^{r-1} + \mu \frac{u^{q-1}}{|x|^s}, & u>0 \quad x \in \Omega, \\ u = 0 & \qquad x \in \partial\Omega \end{cases}$$

where $u \in W_0^{1,p}(\Omega)$, Ω is a bounded domain in $R^N (N > p > 1)$ with smooth boundary $\partial\Omega$ and $0 \in \Omega$, $0 < q < p$, $0 \le s < \frac{N}{p}(p-q) + q$, $p \le r < p^*$, $p^* = \frac{Np}{N-p}$, $\mu > 0$. It reaches the conclusion where this problem has two positive solutions in the different cases . It discusses the existences of positive solutions of the Dirichlet problem for the p-mean curvature operator with supercritical potential firstly. Meanwhile, it extends some results of the p-Laplace operator to that of p-mean curvature operator for $p \ge 2$.

Key Words Mean curvature operator; Mountain Pass Principle; (PS) condition; Ekeland's variational principle.

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1. Introduction

As the elliptic equation with critical potential is widely applied in many majors, especially in physics. So many authors have studied extensively these problems recently. But most of them are for the Laplace or p-Laplacian operator with the convex nonlinearities (see [1-4]).

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For the concave and convex nonlinearities, Antonio Ambroseti, Haim Brezis and Giovanna Cerami [5] researched the following problem in 1994

$$\begin{cases} -\Delta u = \lambda u^q + u^p, & u > 0 \quad x \in \Omega, \\ u = 0 & \quad\quad\quad x \in \partial\Omega \end{cases} \tag{1.1}$$

where $0 < q < 1 < p \leq \frac{N+2}{N-2},\ \lambda > 0$. It reached the conclusion where the above problem had two positive solutions for $\lambda \in (0, \Lambda)$, by using Mini-Max Principle and the method of sub-and supersolutions , where Λ was some positive constant .

Lately, for the concave and convex nonlinearities and critical potential, in 2002, B. Abdellaoui and I. Peral [6] researched the following problem

$$\begin{cases} -\Delta u = \lambda \frac{u^q}{|x|^2} + u^r, & u > 0 \quad x \in \Omega, \\ u = 0 & \quad\quad\quad x \in \partial\Omega \end{cases} \tag{1.2}$$

where $u \in W_0^{1,p}(\Omega),\ \Omega$ is a bounded domain in $R^N (N > p > 1)$ with smooth boundary $\partial\Omega$ and $0 \in \Omega,\ 0 < q < 1 < r < 2^* - 1,\ N \geq 3,\ \lambda > 0$. It reached the same conclusion (see Theorem 2.7) as that of [5] also by using Max-Minimum Principle and the variational methods.

As p-mean curvature operator is useful in geometry, many authors have studied the Dirichlet problem for this operator , such as Shen yaotian([7-9]) has studied the existence of infinitely many solutions for this operator without potential and critical exponents. Resently, Chen zhihui [10] has proved the existence of infinitely many solutions for this operator without potential and critical exponents, by using Mountain Pass Principle with (PSC) conditions. But to my best knowledge, less attention has been given to the solutions of the problems for the p-mean curvature operator with the concave and convex nonlinearities and potential. As lacking the homogeneous property for the p-mean curvature operator, it is difficult to prove that the corresponding functional satisfies Mountain Pass Geometry.

Now we study the case $q < p, \quad r > p$, i.e. the so-called convex-concave case in this paper. In the case of $s = p = 2$, i.e., the potential $|x|^{-2}$ was called critical in [6]. But we think the critical potential is correlate with the potential $\frac{|u|^2}{|x|^2}$. In this paper, as $\frac{N}{p}(p - q) + q > p$, the singularity is higher than that of $p = 2$. So we say that the behaviour of the potential is supercritical.

In this paper, we consider the following problem with supercritical potential

$$\begin{cases} -\mathrm{div}((1 + |\nabla u|^2)^{\frac{p-2}{2}} \nabla u) = \lambda u^{r-1} + \mu \frac{u^{q-1}}{|x|^s}, & u > 0 \quad x \in \Omega, \\ u = 0 & \quad\quad\quad x \in \partial\Omega \end{cases} \tag{1.3}$$

where $u \in W_0^{1,p}(\Omega),\ \Omega$ is a bounded domain in $R^N (N > p > 1)$ with smooth boundary $\partial\Omega$ and $0 \in \Omega,\ 0 < q < p,\ 0 \leq s < \frac{N}{p}(p - q) + q,\ p \leq r < p^*,\ p^* = \frac{Np}{N-p},\ \lambda,\ \mu > 0$. By using Ekeland's variational principle and Max-Minimum Principle and Mountain Pass