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ASYMPTOTICS OF INITIAL BOUNDARY VALUE PROBLEMS OF BIPOLAR HYDRODYNAMIC MODEL FOR SEMICONDUCTORS

Ju Qiangchang
(Academy of Mathematics and System Sciences, CAS,
Beijing 100080, P.R. China)
(E-mail: juqc@mail2.math.ac.cn)
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Abstract In this paper, we study the asymptotic behavior of the solutions to the bipolar hydrodynamic model with Dirichlet boundary conditions. It is shown that the initial boundary problem of the model admits a global smooth solution which decays to the steady state exponentially fast.

Key Words Bipolar hydrodynamic model; semiconductors; asymptotics; smooth solution.

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1. Introduction

We are concerned with the large time behavior of smooth solutions to the one-dimension Euler-Poisson(or hydrodynamic) model for semiconductors in the case of two carriers, i.e. electron and hole

$$n_t + (nu)_x = 0, \tag{1.1}$$

$$h_t + (hv)_x = 0, \tag{1.2}$$

$$(nu)_t + (nu^2 + p(n))_x = n\phi_x - \frac{nu}{\tau_n}, \tag{1.3}$$

$$(hv)_t + (hv^2 + q(h))_x = -h\phi_x - \frac{hv}{\tau_h}, \tag{1.4}$$

$$\phi_{xx} = n - h - d(x), \tag{1.5}$$

$(t, x) \in (0, \infty) \times (0, 1)$ where (n, h) and (u, v) are densities and velocities for electrons and holes, respectively. $j = nu$ and $k = hv$ stand for the electron and hole current densities. ϕ denotes the electrostatic potential and the doping profile $d(x)$ describes fixed charged background ions. τ_n and τ_h are the momentum relaxation times for electrons and holes, respectively. We assume $\tau_n = \tau_h = 1$ for convenience. To simplify the proof, we take $d(x)$ as a nonnegative constant d and choose the typical form for pressure, namely:

$$p(n) = \frac{n^{\gamma_n}}{\gamma_n},\ \gamma_n > 1, \quad q(h) = \frac{h^{\gamma_h}}{\gamma_h},\ \gamma_h > 1. \tag{1.6}$$

The case with two different constants can be dealt with similarly.

Recently, the hydrodynamic model of semiconductors has attracted a lot of attention, because of its function to describe hot electron effects which are not accounted for in the classical drift-diffusion model. Rigorous results have been obtained in various papers. Most of them are concerned with the unipolar case, which only discusses the effect of the electron. However, there are few results on the bipolar case which is of more importance and physical meaning. Fang and Ito [1] show the existence of weak solutions to the system (1.1)-(1.5) in the transonic case using the viscosity argument. Natalini [2], Hsiao and Zhang [3] considered the relaxation limit problem from the bipolar hydrodynamic model to the drift-diffusion equations. Zhu and Hattori [4] showed the existence of the strong solutions to the Cauchy problem of (1.1)-(1.5) and discussed the asymptotic stability of the steady state solution, without the decay rate, when the doping profile is close to zero.

In the present paper, we will consider the initial boundary value problems for (1.1)-(1.5) with the following initial data

$$(n, h, j, k) = (n_0, h_0, j_0, k_0)(x), \quad x \in (0, 1) \tag{1.7}$$

and the density and potential Dirichlet boundary conditions

$$n(0,t) = n(1,t) = \bar{n}, \quad t \geq 0, \tag{1.8}$$

$$h(0,t) = h(1,t) = \bar{h}, \quad t \geq 0, \tag{1.9}$$

$$\phi(0,t) = \phi(1,t) = \bar{\phi}, \quad t \geq 0. \tag{1.10}$$

Here, $\bar{n}, \bar{h}$ and $\bar{\phi} > 0$, and $\bar{n} - \bar{h} = d$. This kind of boundary conditions is commonly used in physics of semiconductor devices.

The goal of this paper is to investigate the global existence of smooth solutions to (1.1)-(1.5) with (1.7)-(1.10) and the large time behavior of them when the initial data (1.7) are assumed to be perturbations of a steady state $(\hat{n}, \hat{h}, \hat{j}, \hat{k}, \hat{E})$ of (1.1)-(1.5) with $\hat{j} = \hat{k} = 0$ which satisfies:

$$\begin{aligned} &p(\hat{n})_x = \hat{n}\hat{\phi}_x, \\ &q(\hat{h})_x = -\hat{h}\hat{\phi}_x, \\ &\hat{\phi}_{xx} = \hat{n} - \hat{h} - d \end{aligned} \tag{1.11}$$

with the boundary condition

$$\begin{aligned} &\hat{n}(0) = \bar{n}, \\ &\hat{h}(0) = \bar{h}, \\ &\phi(0) = \phi(1) = \bar{\phi} \end{aligned}$$