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EXISTENCE OF GLOBAL SMOOTH SOLUTION TO JIN-XIN MODEL WITH LARGE INITIAL DATA*

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Abstract In this paper, Under the assumption that the relaxation time ε is sufficiently small, we prove the existence of the global smooth solution to the Cauchy problem for the Jin-Xin model without any smallness assumption for the initial data. The analysis is based on some a priori estimates which are obtained by the method of characteristic and the maximum principle of first-order quasilinear hyperbolic system.

Key Words Jin-Xin model, maximum principle, global smooth solution.
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1. Introduction

In general case, the solutions to quasilinear hyperbolic systems with dissipative terms only exist in finite time. But, one can get the existence of the global smooth solution. By concerning the above problem, there has been much investigation(cf. [1-14]). At first, Nishida proved in [7] that, for a quasilinear hyperbolic system derived from a quasilinear wave equation, the corresponding Cauchy problem admits a unique global smooth solution provided that the C^1-norm of the initial data is sufficiently small. As an existension of this result to more general case, Hsiao, Li and Qin in [1, 8] discussed the Cauchy problem of the following general first order quasilinear hyperbolic system

$$\frac{\partial u}{\partial t} + \Lambda(u)\frac{\partial u}{\partial x} + f(u) = 0, \tag{1.1}$$

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with initial data

$$u(x,0) = u_0(x). \tag{1.2}$$

Under the assumption that the C^0-norm of the solution is sufficiently small, they proved that the Cauchy problem (1.1), (1.2) admits a unique global smooth solution for $t \geq 0$ provided that the C^1-norm of the initial data is sufficiently small. From their study, it seems that, to guarantee the existence of global smooth solutions to the Cauchy problem of quasilinear hyperbolic systems with dissipative terms, the smallness hypothesis of the C^1-norm of the initial data is necessary. However, Zheng [12] discusses the isentropic Euler equations with damping

$$\begin{cases} v_t - u_x = 0, \\ u_t + p(v,s)_x = -\alpha u, \\ s_t = 0, \end{cases} \tag{1.3}$$

and Yang, Zhu [9] discuss the Cauchy problem for p-system with relaxation

$$\begin{cases} v_t - u_x = 0, \\ u_t + p(v)_x = \dfrac{1}{\varepsilon}\left(f(v) - u\right). \end{cases} \tag{1.4}$$

They prove the existence of the global smooth solution to the corresponding Cauchy problem of (1.3) and the corresponding Cauchy problem of (1.4) under the only assumption that the C^0-norm of the initial data is small, without smallness hypothesis for the C^1-norm respectively. Furthermore, Wang, Li [15] studied the following damped p-system

$$\begin{cases} v_t - u_x = 0, \\ u_t + p(v)_x = -2\alpha u, \\ (v(x,0), u(x,0)) = (u_0(x), v_0(x)), \end{cases} \tag{1.5}$$

and Zhu [13] studied the following typical viscoelastic system with relaxation

$$\begin{cases} v_t - u_x = 0, \\ (\sigma - f(u))_t + \dfrac{\sigma - \mu f(u)}{\delta} = 0, \qquad (\delta > 0, 0 < \mu < 1) \\ (u(x,0), v(x,0)) = (u_0(x), v_0(x)). \end{cases} \tag{1.6}$$

They point out that the Cauchy problem (1.5) and the Cauchy problem (1.6) admit a unique global smooth solution under the assumption that the C^0-norm of the initial data is suitably small, while the C^1-norm of the initial data is not necessarily small respectively.