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MORREY REGULARITY OF SOLUTIONS TO DEGENERATE ELLIPTIC EQUATIONS IN R^n

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Abstract In this paper, we study the Morrey regularity of solutions to the degenerate elliptic equation $-(a_{ij}u_{x_i})_{x_j} = -(f_j)_{x_j}$ in R^n. For this purpose, we introduce four weighted Morrey spaces in R^n.

Key Words Regularity; degenerate elliptic equations; weighted Morrey spaces.
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1. Introduction

The aim of this paper is to consider in R^n the equation

$$Lu \equiv -(a_{ij}u_{x_i})_{x_j} = -(f_j)_{x_j}, \tag{1}$$

for which we assume that $a_{ij}(x)$ are symmetry, measurable and there exists $\nu > 0$, such that for all $\xi \in \mathrm{R}^n$ and a.a. $x \in \mathrm{R}^n$,

$$\nu^{-1}\omega(x)|\xi|^2 \leq a_{ij}(x)\xi_i\xi_j \leq \nu\omega(x)|\xi|^2, \tag{2}$$

where $\omega(x)$ belongs to the Muckenhoupt class A_2. We also assume that $f_j/\omega \in L^2(\mathrm{R}^n, \omega)$.

Since the middle of the 20th century, people have gotten many results about the equation (**??**) in the bounded open subset of R^n. And we can also consider the equation in R^n. In [**?**], S. Leonardi studied in R^n the equation (**??**) in the uniformly elliptic case. We will extend the results in [**?**] to the degenerate case. For this purpose, we will introduce four weighted Morrey spaces in the next section.

2. Preliminaries

We give some definitions first.

Definition 2.1 *Let $\omega(x) > 0$, $\omega(x) \in L^1_{loc}(\mathrm{R^n})$, $1 < p < +\infty$. We say $\omega(x)$ is an A_p weight, which is denoted by $\omega(x) \in A_p$ if*

$$\sup_Q \left(\frac{1}{|Q|}\int_Q \omega(y)dy\right)\left(\frac{1}{|Q|}\int_Q \omega(y)^{-\frac{1}{p-1}}dy\right)^{p-1} \le C < +\infty,$$

where Q is a cube in $\mathrm{R^n}$.

Let ω be an open set of $\mathrm{R^n}$, ω be an A_2 weight, $1 \le p < +\infty$. We give the definitions of weighted Lebesgue spaces and weighted Sobolev spaces.

$L^p(\Omega,\omega)$ is the space of measurable f in Ω, such that

$$\|f\|_{L^p(\Omega,\omega)} = \left(\int_\Omega |f(x)|^p\omega(x)dx\right)^{\frac{1}{p}} < +\infty.$$

$L^\infty(\Omega,\omega)$ is the space of measurable f in Ω, such that

$$\|f\|_{L^\infty(\Omega,\omega)} = \inf\left\{a \ge 0 :\ \omega\left(\{x \in \Omega : |f(x)| > a\}\right) = 0\right\} < +\infty.$$

$Lip(\overline{\Omega})$ denotes the class of Lipschitz functions in $\overline{\Omega}$. $Lip_0(\Omega)$ denotes the class of functions $f \in Lip(\overline{\Omega})$ with compact support contained in Ω. If $f \in Lip(\overline{\Omega})$, we can define the norm

$$\|f\|_{H^{1,p}(\Omega,\omega)} = \|f\|_{L^p(\Omega,\omega)} + \|\nabla f\|_{L^p(\Omega,\omega)}. \tag{3}$$

$H^{1,p}(\Omega,\omega)$ denotes the closure of $Lip(\overline{\Omega})$ under the norm (**??**). We say that $f \in H^{1,p}_{loc}(\Omega,\omega)$ if $f \in H^{1,p}(\Omega',\omega)$ for every $\Omega' \subset\subset \Omega$. $H^{1,p}_0(\Omega,\omega)$ denotes the closure of $Lip_0(\Omega)$ under the norm (**??**).

Now we introduce four kinds of weighted Morrey spaces in $\mathrm{R^n}$. Let $p \ge 1$, $\lambda \in R$, we have

Definition 2.2 *Let* $\|f\|^p_{L^{p,\lambda}(\mathrm{R^n},\omega)} = \sup\limits_{\substack{x\in\mathrm{R^n}\\ r>0}} \frac{r^{-\lambda}}{\omega(B_r(x))}\int_{B_r(x)} |f(y)|^p\omega(y)dy$. *We set*

$$L^{p,\lambda}(\mathrm{R^n},\omega) = \left\{f \in L^p(\mathrm{R^n},\omega) :\ \|f\|_{L^{p,\lambda}(\mathrm{R^n},\omega)} < +\infty\right\}.$$

Definition 2.3 *Let* $\|f\|^p_{\tilde{L}^{p,\lambda}(\mathrm{R^n},\omega)} = \sup\limits_{\substack{x\in\mathrm{R^n}\\ r>0}} \omega(B_r(x))^{-\lambda}\int_{B_r(x)} |f(y)|^p\omega(y)dy$. *We set*

$$\tilde{L}^{p,\lambda}(\mathrm{R^n},\omega) = \left\{f \in L^p(\mathrm{R^n},\omega) :\ \|f\|_{\tilde{L}^{p,\lambda}(\mathrm{R^n},\omega)} < +\infty\right\}.$$

Definition 2.4 *Let* $\|f\|^p_{M^{p,\lambda}(\mathrm{R^n},\omega)} = \sup\limits_{x\in\mathrm{R^n}} \int_0^{+\infty} r^{-\lambda-1}\left(\int_{B_r(x)} |f(y)|^p\omega(y)dy\right)dr$. *We set*

$$M^{p,\lambda}(\mathrm{R^n},\omega) = \left\{f \in L^p(\mathrm{R^n},\omega) :\ \|f\|_{M^{p,\lambda}(\mathrm{R^n},\omega)} < +\infty\right\}.$$