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INITIAL-BOUNDARY VALUE PROBLEM FOR THE LANDAU-LIFSHITZ SYSTEM WITH APPLIED FIELD

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Abstract In this paper, the existence and partial regularity of weak solution to the initial-boundary value problem of Landau-Lifshitz equations with applied fields in a 2D bounded domain are obtained by the penalty method.
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1. Introduction

The Landau-Lifshitz (LL) system which describes the evolution of spin fields in continuum ferromegnets bears a fundamental role in the understanding of nonequilibrium magnetism, just as the Navier-Stokes equation does in that of fluid dynamics. The LL system for a spin chain with an easy plane

$$u_t = u \times u_{xx} + u \times Ju$$

has been studied by the inverse scattering method in [1–3] where $u = (u^1, u^2, u^3)$ is the spin vector, $J = \mathrm{diag}\,\{J_1, J_2, J_3\}$ with $J_1 \leq J_2 \leq J_3$ and "$\times$" denotes the vector cross product in R^3. More general LL system of the following form

$$u_t = u \times F_{\text{eff}} - \lambda u \times (u \times F_{\text{eff}})$$

was also studied in [4] where $F_{\text{eff}} = \nabla^2 u - 2A(u \cdot n)n + \mu B$, $n = (0, 0, 1)$, A is the anisotropy paramenter ($A > 0$, easy plane; $A < 0$, easy axis), μ is the gyromagnetic ration in Bohr magnetons, λ is the Gilbert damping constant and $\boldsymbol{B}$ is the external magnetic field.

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A lot of works contributed to the study of solutions to the Landau-Lifshitz systems

$$u_t = -\alpha_1 u \times (u \times \Delta u) + \alpha_2 u \times \Delta u$$

of 1- or 2-dimensional spin chain motion have been made by mathematicians.

In 1993, Guo and Hong [5] established the global existence and partial regularity theorems concerning the weak solutions from a 2-dimensional Riemannian manifold (without boundary) into the unit sphere S^2 with standard metric and revealed the links between the solutions and the harmonic maps. They found that the solutions have the same partial regularity as that of the harmonic map heat flow [6]. Also for $m = 2$, the uniqueness of weak solution to the initial problem satisfying energy inequality can be found in [7]. The conclusions of [5] were extended to a class of generalized Landau-lifshitz system in [8].

The existence and partial regularity results for the weak solution of the nonhomogeneous initial-boundary value problem with $m = 2$ (without applied fields) were established in [9] in which the authors introduced a method much different from before which originates from the study of Ginzburg-Landau functional [10].

In this paper, we let $\Omega \subset R^n$ $(n = 1, 2)$ be a bounded smooth domain and consider the following nonhomogeneous initial-boundary value problem

$$u_t = -u \times (u \times \Delta u) + u \times \Delta u + u \times \boldsymbol{H}(u, x, t), \quad \text{in } \Omega \times R_+ \tag{1.1}$$

$$u|_{\partial\Omega \times R_+} = \psi(x), \quad u|_{\Omega \times \{t=0\}} = \varphi(x), \quad |\varphi(x)| = 1 \tag{1.2}$$

which is a natural general form including both "easy plane" and the external field.

Because of the action of the external field $\boldsymbol{H}$, one can not expect to get the smoothness away from a set consisting of only finitely many points as for the case $\boldsymbol{H} \equiv 0$. We can only get the smoothness away from at most countably many lines in $\overline{\Omega} \times [0, \infty)$.

Our main results are Theorem 4.1 (existence and partial regularity) and Theorem 5.2 (smooth solution of 1-D problem). In this paper we denote $\Omega(t) = \Omega \times \{t\}$, $\Omega_t = \Omega \times (0, t)$, $B_r(x)$ the disk centered at x with radius r.

2. A Penalty Problem and Weak Solution to (2.1)–(1.2)

Since $|\varphi(x)| = 1$ on $\overline{\Omega}$, it is easy to verify that $|u(x, t)| \equiv 1$ and it follows from [5] that u is a solution of (1.1)–(1.2) if and only if u is a solution in the classical sense of the following system

$$\frac{1}{2}u_t - \frac{1}{2}u \times u_t = \Delta u + u|\nabla u|^2 + \frac{1}{2}u \times \boldsymbol{H} - \frac{1}{2}u \times (u \times \boldsymbol{H})$$

Therefore, it is natural to consider the following equation

$$\frac{1}{2}u_t - \frac{1}{2}u \times u_t = \Delta u + u|\nabla u|^2 + u \times \boldsymbol{f}(u, x, t) \tag{2.1}$$