J.Partial Diff. Eqs. **13**(2000), 151–162
©International Academic Publishers Vol.13, No.2

CHARPIT SYSTEM OF PARTIAL DIFFERENTIAL EQUATIONS WITH ALGEBRAIC CONSTRAINTS

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(Received Feb. 12, 1999)

Abstract In this paper the Charpit system of partial differential equations with algebraic constraints is considered. So, first the compatibility conditions of a system of algebraic equations and also of the Charpit system of partial differential equations are separately considered. For the combined system of equations of both types sufficient conditions for the existence of a solution are found. They lead to an algorithm for reducing the combined system to a Charpit system of partial differential equations of dimension less than the initial system and without algebraic constraints. Moreover, it is proved that this system identically satisfies the compatibility conditions if so does the initial system.

Key Words Charpit differential equations; system of partial differential equations; algebraic constraints.
1991 MR Subject Classification 35F20.
Chinese Library Classification O175.22, O175.29.

1. Compatibility Conditions for a System of Algebraic Equations

We assume throughout this paper that all functions are of sufficiently high smoothness class C^r. All the results have a local character.

At first we will consider the compatibility conditions for a system of equations of the form

$$f(z) = 0 \tag{1.1}$$

where z is a vector of unknowns and f is a given vector function.

* The author supported partially by the Bulgarian Science Fund under Grant MM-706.

Let us rearrange the components of the vector function f into two subvectors $\overline{f}$ and $\widetilde{f}$ such that the rows of the matrix $\partial\overline{f}/\partial z$ are linearly independent and the rows of the matrix $\partial\widetilde{f}/\partial z$ are dependent on the rows of the previous matrix, i.e. there exists a matrix $\mathbf{V}$ such that

$$\frac{\partial\widetilde{f}}{\partial z}(z) = \mathbf{V}(z)\cdot\frac{\partial\overline{f}}{\partial z}(z) \tag{1.2}$$

This means that there exists a partition (and eventual permutation) of the components of the vector z into subvectors z' and z'' such that the matrix

$$\frac{\partial\overline{f}}{\partial z'}(z)$$

is nonsingular. Thus the system of the equation (1.1) decomposes into two systems:

$$\overline{f}(z', z'') = 0 \tag{1.3a}$$

$$\widetilde{f}(z', z'') = 0 \tag{1.3b}$$

From the implicit function theorem [1] it follows that if there exists a point z_0 such that $\overline{f}(z_0) = 0$, then in a neighbourhood of z_0 there exists a solution z' of (1.3a) in the following form

$$z' = \phi(z'') \tag{1.4}$$

where z'' is arbitrary parameter. Substituting this solution into (1.3b), we will prove that it takes a value independent of z'', i.e.

$$\frac{\partial}{\partial z''}\widetilde{f}(\phi(z''), z'') = 0 \tag{1.5}$$

By the differentiation of (1.3a) we obtain

$$\frac{\partial\overline{f}}{\partial z''} + \frac{\partial\overline{f}}{\partial z'}\cdot\frac{\partial\phi}{\partial z''} = 0 \quad\Rightarrow\quad \frac{\partial\phi}{\partial z''} = -\Big(\frac{\partial\overline{f}}{\partial z'}\Big)^{-1}\cdot\frac{\partial\overline{f}}{\partial z''} \tag{1.6}$$

On the other side, the condition (1.2) decomposes into the following two systems

$$\frac{\partial\widetilde{f}}{\partial z'} = \mathbf{V}\cdot\frac{\partial\overline{f}}{\partial z'}, \quad \frac{\partial\widetilde{f}}{\partial z''} = \mathbf{V}\cdot\frac{\partial\overline{f}}{\partial z''} \tag{1.7}$$

Next we prove (1.5)

$$\begin{aligned}\frac{\partial}{\partial z''}\widetilde{f}(\phi(z''), z'') &= \frac{\partial\widetilde{f}}{\partial z''} + \frac{\partial\widetilde{f}}{\partial z'}\cdot\frac{\partial\phi}{\partial z''} = \frac{\partial\widetilde{f}}{\partial z''} - \frac{\partial\widetilde{f}}{\partial z'}\cdot\Big(\frac{\partial\overline{f}}{\partial z'}\Big)^{-1}\cdot\frac{\partial\overline{f}}{\partial z''}\\ &= \mathbf{V}\cdot\frac{\partial\overline{f}}{\partial z''} - \mathbf{V}\cdot\frac{\partial\overline{f}}{\partial z'}\cdot\Big(\frac{\partial\overline{f}}{\partial z'}\Big)^{-1}\cdot\frac{\partial\overline{f}}{\partial z''} = 0\end{aligned}$$

Thus we prove the following theorem.

Theorem 1.1 *If there exists a point z_0 such that $\overline{f}(z_0) = 0$, then the validity of the equality*

$$\widetilde{f}(z_p) = 0 \tag{1.8}$$