J. Partial Diff. Eqs. **13**(2000), 301–319
©International Academic Publishers Vol.13, No.4

EVOLUTION OF NONPARAMETRIC SURFACES WITH SPEED DEPENDING ON CURVATURE FUNCTION*

Liu Huizhao and Wang Yan
(Department of Applied Mathematics, Hebei University of Technology, Tianjin 300130, China)
Wang Guanglie
(Department of Mathematics, Jilin University, Changchun 130023, China)
(Received Dec. 11, 1998; revised June 12, 2000)

Abstract We investigate the asymptotic behaviour of solution of the initial-boundary value problem for the equation, which describes the evolution of graphs with speed depending on curvature function of current graphs. In contrast with [1-5] and others, in which the speed of flow of graphs is directly proportional to the curvature of graphs, here we discuss is that the speed is inversely proportional to the curvature of graphs, and our methods is different from theirs.
Key Words Evolution of surface; curvature function; boundary condition; classical solution; asymptotic behaviour.
1991 MR Subject Classification 35K55, 53A05.
Chinese Library Classification O175.29.

1. Introduction and Main Result

Let Ω be a bounded domain in R^n with smooth strictly convex boundary $\partial\Omega$, and let $u(x,t), x \in \bar{\Omega}, t \in [0,\infty)$ is a solution of initial-boundary value problem

$$-D_t u = \frac{\psi(x)}{f(\kappa(u))} > 0, \qquad \forall (x,t) \in Q = \Omega \times (0,\infty] \tag{1.1}$$

$$u(x,t) = -\nu(t) < 0, \qquad \forall (x,t) \in \partial\Omega \times [0,\infty) \tag{1.2}$$

$$u(x,t) = \mu(x) \le 0, \qquad \forall (x,t) \in \bar{\Omega} \times \{t=0\} \tag{1.3}$$

where $\psi(x) \in C^{2+\alpha}(\bar{\Omega})$ is a given positive function on $\bar{\Omega}$, $\kappa(u) = (\kappa_1(u), \kappa_2(u), \cdots, \kappa_n(u))$ represents the principal curvatures of the graph $(x, u(x,t))$ when t is viewed as a parameter, and, as in [6], the function f is a smooth symmetric function defined in an symmetric open convex cone $\Gamma \subset R^n$ with vertex at origin. Γ is assumed to be symmetric in κ_i, containing the positive cone $\Gamma^+ = \{x \in R^n | x_i > 0, i = 1, 2, \cdots, n\}$, and f is required to satisfy the following condition:

$$f(\kappa) > 0, \ \frac{\partial f}{\partial \kappa_i}(\kappa) > 0, \quad i = 1, 2, \cdots, n, \quad \kappa \in \Gamma \tag{1.4}$$

* The work supported by National Science Foundation of China and the Doctor Foundations of Hebei Education Commission.

$$f \text{ is a concave function} \tag{1.5}$$

Geometrically, the graphs $(x, u(x,t))$ may be viewed as a family of (hyper-) surfaces expanding in R^{n+1} whose relative boundaries $(x, v(x,t))|_{\partial\Omega}$ remain moved.

Curvature flow with boundary condition has been studied by many authors, for details see [1-5] and reference therein. In contrast to these papers, in which the speed of flow of graphs is directly proportional to the curvature of graphs, here we discuss is that the speed is inversely proportional to the curvature of graphs, and our methods is different from theirs.

It should be pointed out that the closed surfaces expansion, without boundary conditions, have been studied by many authors [7-11]. Our method is totally different from theirs, for the details, see [7-11] and the references therein.

The main objective of this note is to investigate the asymptotic properties of classical solution of (1.1)–(1.3). Appropriate existence result for (1.1)–(1.3) has been established in [12] with the help of the results in [6] [13].

The asymptotic property of solution (1.1)–(1.3) is analyzed in three steps. In the special case: $\psi(x) \equiv p, \nu(t) = at$, and Ω is a ball B_r, we give a sharp estimate and, by which, we know that the surfaces tend to the lower hemisphere. In other two cases, we show that the surfaces tend to the surface $(x, w(x))$, where $w(x)$ is given by the problem

$$f(\kappa(w)) = \frac{\psi(x)}{\nu_0}, \quad x \in \Omega \tag{1.6}$$

$$w(x) = 0, \quad x \in \partial\Omega \tag{1.7}$$

where $\lim\limits_{t\to\infty} D_t\nu(t) = \nu_0 > \nu_m > 0, \nu(t)$ comes from (1.2).

As [6] and [12], we call $w(x) \in C^2(\bar\Omega)$, or $u(x,t) \in C^{2,1}_{\text{loc}}(\bar\Omega)$ is *admissible* if it belong to *admissible set*

$$E = \{w(x)|\kappa(w(x)) \in \Gamma, \quad \forall x \in \Omega\}$$

or

$$P = \left\{u(x,t)|\kappa(u(x,t)) \in \Gamma; \quad \frac{\partial u(x,t)}{\partial t} < 0 \text{ for } t \in [0,\infty), \text{ and } \forall x \in \Omega\right\}$$

From [6] and [12], we know that (1.6) and (1.1) is, respectively, elliptic equation and parabolic equation in admissible function class.

In this paper, our discussion is always in admissible function class.

The problem (1.6)–(1.7) has been studied in [6]. In order to use some results in [6] directly, as in [6], besides (1.4)–(1.5) we assume that

(I)

$$\overline{\lim_{\kappa\to\kappa_0}} f(\kappa) = 0 \quad \text{for every} \quad \kappa_0 \in \partial\Gamma \tag{1.8}$$

(II) For every $C > 0$ and every compact set K in Γ there is a number $R = R(C,K)$ such that

$$f(\kappa_1, \kappa_2, \cdots \kappa_{n-1}, \kappa_n + R) \geq C, \quad \kappa = (\kappa_1, \kappa_2, \cdots, \kappa_n) \in K \tag{1.9}$$