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A NOTE ON SOLVABILITY OF THE NONLINEAR ABSTRACT VISCOELASTIC PROBLEM IN BANACH SPACES*

Doherty Andrade
(Universidade Estadual de Maringá -doherty@dma.uem.br. Departamento de Matemática - 87020-900 Maringá-PR, Brazil)
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Abstract In this work we are concerned with the existence of integral solution for a nonlinear abstract viscoelastic problem in Banach space, where the operator is accretive dependent on time.
Key Words Accretive operator; integral Solution; viscoelasticity.
Classification 47H15, 47H04, 47H06.

1. Introduction

In this work we are concerned with the solvability of the nonlinear abstract problem in viscoelasticity with memory,

$$\begin{cases} \dfrac{du}{dt} + A(t)u(t) \ni \displaystyle\int_0^t k(t-s)g(s,u(s))ds, \quad 0 \le t \le T \\ u(0) = x \in D = \overline{D(A(0))} \end{cases} \tag{1.1}$$

where the family $A(t) : D((A(t)) \subseteq X \to 2^X$, $0 \le t \le T$ is at the least m-accretive operator in Banach space X, $g : [0,T] \times D \to X$ and $k : [0,T] \to L(X)$ are continuous functions. Problems of this type appear in various applied areas and several authors have studied different versions of (1.1). In particular, we note that more general mathematical model of heat conduction in a beam or vibrations of a string both with memory can be rewritten as an integro-differential equation with inclusion. There are another examples as in thermoelastic evolution systems. See [1], [2] and [3].

There are several works and methods on solvability of functional evolution equations with accretive operators dependent on t but as far as we know nothing on viscoelasticity system. The main goal of this work is to prove the existence of integral solution of (1.1). We use the fixed point theorem and auxiliary results to obtain the existence of integral solution. The idea of the proof is to define an operator P where its fixed points are integral solutions.

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In what follows, by X we denote a real Banach space with norm $\|\cdot\|$ and dual X^*, J is the duality mapping. We recall that

$$\langle y,x\rangle_s = \lim_{h\to 0^+} \frac{\|x+hy\|^2-\|x\|^2}{2h}$$

$$\langle y,x\rangle_i = \lim_{h\to 0^-} \frac{\|x+hy\|^2-\|x\|^2}{2h}$$

We know that for every $x, y \in X$ there exists $x_1^*, x_2^* \in Jx$ such that

$$\langle y,x\rangle_s = x_1^*(y) = \sup\{x^*(y), x^* \in Jx\}$$

$$\langle y,x\rangle_i = x_2^*(y) = \inf\{x^*(y), x^* \in Jx\}$$

We note that $\langle,\rangle_s$ is upper semicontinuous and $\langle,\rangle_i$ is lower semicontinuous. For these facts and additional properties of these directional derivatives please see [1] and [4].

Given a set valued operator A we denote by $D(A)$ and $R(A)$ the sets $\{x \in X, Ax = \emptyset\}$ and $\bigcup_{x\in D(A)} Ax$, respectively. For simplicity, set valued operators are identified with their graphs, i.e., if $y \in Ax$ we denote by $(x,y) \in A$. An operator $A: D(A) \subseteq X \to 2^X$ is called accretive if only if for every $\lambda \in \mathbb{R}^+$ and every $y_1 \in Ax_1, y_2 \in Ax_2$, we have

$$\|x_1 - x_2\| \le \|(x_1 - x_2) + \lambda(y_1 - y_2)\|$$

Equivalently, A is accretive if for every $x_1, x_2 \in D(A)$ there exists $j \in J(x_1 - x_2)$ such that $\langle y_1 - y_2, j\rangle \ge 0$, for all $y_1 \in Ax_1$ and $y_2 \in Ax_2$.

An accretive operator is called m-accretive if $R(I + \lambda A) = X$, for every $\lambda \in \mathbb{R}^+$, here I is the identity mapping.

To introduce a notion of solution we consider the functional problem

$$\begin{cases} u'(t) + A(t)u(t) \ni f(t), & t \in [0,T] \\ u(0) = x_0 \end{cases} \qquad (A^f_{x_0})$$

where $A(t)$ is a set valued nonlinear accretive operator and $f \in C([0,T];X)$.

Definition 1.1 *We say that $u : [0,T] \to X$ is a strong solution of $(A^f_{x_0})$ if it is absolutely continuous on compact subsets of $(0,T)$, strongly differentiable on $(0,T)$ and satisfies $(A^f_{x_0})$ almost every on $(0,T)$.*

Next we introduce the so called discrete approximate solution. Let $0 = t_{n,0} < t_{n,1} < \cdots < t_{n,N(n)} = T$ be a partition such that

$$\max_{1\le k\le N(n)} (t_{n,k} - t_{n,k-1}) \to 0$$

as $n \to \infty$. Suppose that there exists $\{x_{n,k}\} \subset X$ with $x_{n,k} \in D(A(t_{n,k}))$ and

$$f_{n,k} - \frac{x_{n,k} - x_{n,k-1}}{t_{n,k} - t_{n,k-1}} \in A(t_{n,k})x_{n,k}$$