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WELL-POSEDNESS OF CAUCHY PROBLEM FOR COUPLED SYSTEM OF LONG-SHORT WAVE EQUATIONS*

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Dedicated to Professor Ding Xiaxi on the occasion of his 70th birthday

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Abstract In this paper we study the Cauchy problem for a class of coupled equations which describe the resonant interaction between long wave and short wave. The global well-posedness of the problem is established in space $H^{\frac{1}{2}+k} \times H^k$ ($k \in Z^+ \cup \{0\}$), the first and second components of which correspond to the short and long wave respectively.

Key Words Cauchy problem, long-short wave equation, well-posedness
Classification 35Q30, 35G25.

1. Introduction and Main Results

In this note we study Cauchy problem for the following long-short wave equation

$$iu_t + u_{xx} = uv + a|u|^{p-1}u, \quad (t, x) \in \mathbf{R} \times \mathbf{R} \tag{1.1}$$

$$v_t = (|u|^2)_x, \quad (t, x) \in \mathbf{R} \times \mathbf{R}, \tag{1.2}$$

$$u(0) = u_0(x), \quad x \in \mathbf{R} \tag{1.3}$$

$$v(0) = v_0(x), \quad x \in \mathbf{R} \tag{1.4}$$

where $a \in \mathbf{R}$, $u(t, x)$ and $v(t, x)$ represent the envelope of the short wave and the amplitude of the long wave respectively. The equations (1.1) (1.2) arise in the study of surface waves with both gravity and capillary modes present and also in plasma physics[1,2]. For $a = 0$, Ma[3] studied (1.1)–(1.4) by inverse scattering method under

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suitable smooth conditions on initial functions. Concerning the Cauchy problem (1.1)–(1.4) in usual Sobolev spaces for (u, v), Guo[4] first proved the global solvability of (1.1)–(1.4) in space $L^\infty(0,T;H^m) \times L^\infty(0,T;H^m)$ for all integer $m \geq 2$ by means of integral estimation method and the fixed point theorem when $a = 0$. Recently Tsutsumi & Hatano[5] proved the following results:

(i) When $a = 0$, $u_0 \in H^{\frac{1}{2}}$, $v_0 \in L^2 \cap L^\infty$, they prove global solvability of (1.1)–(1.4) in the Space $H^{\frac{1}{2}} \times L^2$.

(ii) When $a \neq 0$, $p = 3$, they proved the global well-posedness in space $H^{\frac{1}{2}+k} \times H^k$ for all integers $k \geq 1$.

One natural problem is whether (1.1)–(1.4) generates global flow in the space $H^{\frac{1}{2}} \times L^2$ (or $H^{\frac{1}{2}+k} \times H^k$ for all integer $k \geq 1$) for general p. Our purpose here is to study the global well-posedness of (1.1)–(1.4) in the space $H^{\frac{1}{2}+k} \times H^k$ $(k \in Z^+ \cup \{0\})$ for general $p \geq 2$. Our main tools are so called Strichartz type estimates which were established in [6–8] and contraction mapping principle.

Before we state our results we first introduce several notations. For $1 \leq p \leq \infty$, we denote by $L^p(\mathbf{R})$ usual Lebegue space of complex and real value functions. $J_{-s} = (I - \Delta)^{-\frac{s}{2}}$ denotes usual Bessel potential, we denote by $W^{s,p}(\mathbf{R}) = J_{-s}L^p$ Bessel potential space. When s is an integer, $W^{s,p}$ is just usual Sobolev space. In particular, we simplify write $W^{s,2} = H^s$. Let $D_x^s = (-\Delta)^{\frac{s}{2}}$, then D^{-s} denotes Riesz potential, we denote by $\dot{W}^{s,p}(\mathbf{R}) = D^{-s}L^p$ Riesz potential space. For a Banach space X and a time interval $I \in \mathbf{R}$, we denote by $C(I, X)$ the space of strong continuous function from I to X and by $L^p(I, X)$ the space of measurable functions u from I to X such that $\|u(\cdot)\|_X \in L^p(I)$. For the sake of convenience we usually write $L_t^q L_x^p = L^q(I, L^p(\mathbf{R}))$ and $L_x^p L_t^q = L^p(\mathbf{R}, L^q(I))$ when this causes no confusion. Different positive constant in the estimates blows might be denoted by the same letter C and if necessary by $C(*, \cdots, *)$ in order to indicate the dependence on the quantities appearing in parentheses.

As is standard practice, we study (1.1)–(1.4) via the corresponding integral equations

$$u(t) = S(t)u_0(x) + \int_0^t S(t-\tau)(uv + a|u|^{p-1}u)d\tau, \tag{1.5}$$

$$v(t) = v_0(x) + \int_0^t \partial_x |u|^2 ds \tag{1.6}$$

where $S(t) = \exp(it\Delta)$ is the free propagator which solves free Schrödinger equation.