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ATTRACTORS FOR THE LONG-SHORT WAVE EQUATIONS

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Abstract In the present paper, we study the long time behaviour of solutions for the long-short wave equations with zero order dissipation. We first construct the global weak attractor for this system in $H^2_{per} \times H^1_{per}$. And then by exact analysis of two energy equations, we show that the global weak attractor is actually the global strong attractor in H^2_{per}.

Key Words Global attractor; zero order dissipation; long-short wave equations.
Classification 35K57, 35B40, 35P10.

1. Introduction

The long-short wave equations

$$iS_t + S_{xx} - LS = 0 \tag{1.1}$$
$$L_t + |S|^2_x = 0 \tag{1.2}$$

are first derived by Djordjevic and Redekopp in [1], where S is the envelope of the short wave, L is amplitude of long wave and is real. This system describes the resonance interaction between the long wave and the short wave. As pointed out in [1], the physical significance of this system is such that the dispersion of the short wave is balanced by nonlinear interaction of the long wave with short wave, while the evolution of the long wave is driven by the self-interaction of the short wave. This system also appears in analysis of internal wave in [2], as well as Rossby waves. Denney presents a general theory for the interaction between the short wave and the long wave in [3], Guo obtains the existence of global solution for long-short wave equations and generalized long-short wave equations in [4] and [5] respectively. The orbital stability of solitary waves for this system have been studied in [12].

In this paper, we investigate the asymptotic behaviour of solutions for the following long-short wave equations with zero order dissipation:

$$iu_t + u_{xx} - nu + i\alpha u + g(|u|^2)u + h_1(x) = 0 \tag{1.3}$$

$$n_t + |u|_x^2 + \delta n + f(|u|^2) + h_2(x) = 0 \tag{1.4}$$

where α and δ are positive constants. Our aim here is to derive the existence of the global attractor for the system (1.3)–(1.4). Since the nonlinear semigroup $S(t)$ generated by (1.3)–(1.4) is not compact in $H^2_{\rm per}$, we cannot construct the global attractor by the method introduced by Temam [6] or Constantin, Foias and Temam [7]. We here first apply the techniques developed by Ghidaglia [8] to show the existence of the global weak attractor for the system (1.3)–(1.4) in $H^2_{\rm per}$. For that purpose, it is necessary that the semigroup $S(t)$ should be weakly continuous in $H^2_{\rm per}$ for every $t > 0$, which is obtained in [8] by the continuity of $S(t)$ in $H^1_{\rm per}$ and $H^2_{\rm per}$. However, in our case, because of the interaction of the equation (1.3) and (1.4), we cannot derive the continuity of $S(t)$ in $H^1_{\rm per}$. And hence the weak continuity of $S(t)$ is not obtained by the method in [8]. We here employ a direct method to establish the weak continuity. After we have shown the existence of the global weak attractor, by two energy equations and an idea of Ball [9] we conclude that the global weak attractor is actually the global strong attractor for $S(t)$ in $H^2_{\rm per}$. Since we cannot prove the uniform differentiability of $S(t)$ on the attractor, the problem whether the dimensions of the global attractor are finite or not remains open. We think the dimensions should be finite, but we are unable to prove it.

This paper is organized as follows. In Section 2, we recall some facts about the solution semigroup $S(t)$. By a direct method we show that $S(t)$ is weakly continuous in $H^2_{\rm per}$ for every $t > 0$. Section 3 contains uniform *a priori* estimates in time. In particular, we obtain that the dynamical system $S(t)$ possesses a bounded absorbing set in $H^2_{\rm per}$. In Section 4, we first establish the existence of the global weak attractor, then by energy equations we prove that the global weak attractor is actually the global strong attractor in $H^2_{\rm per}$.

2. The Solution Semigroup

In this section, we consider the following generalized nonlinear wave equations:

$$iu_t + u_{xx} - nu + i\alpha u + g(|u|^2)u + h_1(x) = 0 \tag{2.1}$$

$$n_t + |u|_x^2 + \delta n + f(|u|^2) + h_2(x) = 0 \tag{2.2}$$