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EXISTENCE OF TRAVELLING WAVE SOLUTION OF NONLINEAR EQUATIONS WITH NONLOCAL ADVECTION*

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Dedicated to Professor Gu Chaohao on the occasion of his 70th birthday and his 50th year of educational work

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Abstract In this paper, the existence of travelling wave solution for nonlinear equation with nonlocal advection

$$\rho\frac{\partial}{\partial t}\left(\frac{u^m}{m}\right)=\frac{\partial^2 u}{\partial x^2}-\frac{\partial}{\partial x}[\varphi(k*u)u]+u^n f(u)$$

is studied in the case of $m\geq 1$, $n\geq 1$. When ε,φ,f,m and n satisfy some determinate conditions, there exists the travelling wave solution.

Key Words Travelling wave solution; nonlocal advection.
Classification 35K55, 35K27.

1. Introduction

In this paper, we are concerned with the existence of travelling wave solution of nonlinear equation with nonlocal advection

$$\rho\frac{\partial}{\partial t}\left(\frac{u^m}{m}\right)=\frac{\partial^2 u}{\partial x^2}-\frac{\partial}{\partial x}[\varphi(k*u)u]+u^n f(u) \tag{1.1}$$

where $m\geq 1$, $n\geq 1$ and $\rho=\operatorname{sgn} u_x$. Here $u(t,x)\geq 0$ is the population density at the time $t>0$ and position $x\in R$, $\varphi(s)$ is the velocity of population. We assume that $\varphi(s)$ ($s\in R$) is a strict monotone increased upper convex odd function. The second term on the right-hand side of Eq. (1.1) involved nonlocal advective term. One of the simplest examples of K is

$$K=\varepsilon[1-2H(x)],\quad \varepsilon\geq 0 \tag{1.2}$$

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where $H(x)$ is the Heaviside step function. The nonlocal advective term has been discussed by Mimura [1] and Alt [2] from a view of biological aggregation. Because of the advection, the individuals have the tendency of aggregation. The third term on the right-hand side of (1.1) is a kinetics of the process, which represents the supply due to births and deaths. We assume that $f(s)$ is a function satisfying

$$f \in C^1[0,1],\ f(0) < 0,\ f(1) = 0,\ f'(1) < 0,\ f(x) = \begin{cases} < 0, & u \in (0,a) \\ > 0, & u \in (a,1) \end{cases} \tag{1.3}$$

generally speaking, $f(s) \sim (s-a)(1-s)$. Ecological studies of this nonlinearity are discussed by Okubo[3]. The discrete model for spatially aggregation phenomena for nonlinear degenerate diffusion equation involving a nonlocal advection term is investigated by Ikeda[4]. At present, the studies of travelling wave solution of reaction diffution equation are quite ripe[5].

For $m = n = 1$, the existence of travelling wave solution of (1.1) was investigated by Huang Sixun[6] in the three dimensions, by using the methods to deal with travelling wave solution of reaction diffution equation and techniques to deal with stationary solution. In this paper, we discuss the existence of travelling wave solution of (1.1), by using the techniques of [7] and the techniques to deal with nonlinear degenerate equation.

2. Mathematical Model

In this paper, we consider travelling wave solution of Eq. (1.1), when K takes the form of (1.2). Then $k * u$ can be represented by

$$k * u = \varepsilon\Big[-2\int_{-\infty}^{x} u(t,y)dy + I\Big] \tag{2.1}$$

where $I = \int_{-\infty}^{+\infty} u(t,y)dy$, generally speaking, $I = I(t)$. Set

$$v(t,x) = \int_{-\infty}^{x} u(t,y)dy, \quad u_x(t,x) = w(t,x) \tag{2.2}$$

then $v_x(t,x) = u(t,x)$. Substituting above relation forms into (1.1), we have

$$\rho\frac{\partial}{\partial t}\Big(\frac{u^m}{m}\Big) = \frac{\partial^2 u}{\partial x^2} + 2\varepsilon\dot{\varphi}[\varepsilon(2v - I)]u^2 + \varphi[\varepsilon(2v-I)]w + u^n f(u) \tag{2.3}$$

where $\dot{\varphi} = \dfrac{d\varphi}{ds} > 0$. Set

$$u(t,x) = u(\theta), v(t,x) = v(\theta), \quad w(t,x) = w(\theta), \quad \theta = x + ct \tag{2.4}$$