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MULTILEVEL ITERATIVE ALGORITHMS FOR ACCELERATING SPECTRAL APPROXIMATIONS OF INTEGRAL EQUATIONS*

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Dedicated to Professor Ding Xiaxi on the Occassion of His 70th Birthday

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Abstract In this paper, spectral collocation and Nyström methods for integral equations are discussed. Some multilevel correction schemes are presented for infinitely accelerating the covergence even if the exact solution is not so smooth.

Key Words Integral equation; iterative correction; spectral collocation and Nyström methods

Classification 65R20.

1. Introduction

The spectral method is an approach for high accuracy approximation when the exact solution is smooth. The first serious application of the spectral method to partial differential equations was due to Silberman [1]. We refer to Bernardi and Maday [2, 3], Canuto, Hussaini, Quarteroni and Zang [4] and Guo [5] for the development and the state of art in theory and application of spectral methods to partial differential equations, and to Golberg [6–9] and Solan and Burn [10] for the applications to the integral equations.

For accelerating approximations of integral equations, a multilevel iterative correction algorithm has been recently proposed, see Lin and Zhou [11]. This algorithm is of special interest in the case where the kernel is of finite smoothness and that the spectral appoximation itself is of less accuracy. In such a case the iterative correction can be used to infinitely accelerate the convergence order for spectral approximation even if the exact solution is not smooth (in L^2 for example). Such interesting results, i.e., the finite smoothness in the kernel implies the infinite accuracy in approximation, do not hold for partial differential equations.

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In this paper, we shall show such a multilevel iterative algorithm is also valid for spectral collocation and Nyström methods. The outline of this paper is as follows: In the coming section, some preliminaries are presented. In Section 3 and Section 4, spectral collocation and discrete spectral collocation together with the spectral Nyström methods are discussed. In Section 5, a spectral Kantorovich-collocation scheme is proposed for nonsmooth solutions. Finally, some conclusional remarks are added.

2. Preliminaries

Consider the integral equation

$$u(x)+\int_{-1}^{1}k(x,y)u(y)dy=f(x)\quad \text{in}[-1,1] \tag{1}$$

where f is given and $k(x,y)\in L^2[-1,1]^2$.

Let K be the operator defined by

$$(Ku)(x)=\int_{-1}^{1}k(x,y)u(y)dy$$

then (1) can be written as

$$(I+K)u=f \tag{2}$$

where I is the identical operator.

We assume that from $L^2[-1,1]$ into $L^2[-1,1]$, $(I+K)^{-1}$ exists and is bounded. One sees that if $k\in C^r[-1,1]^2$, then

$$\|(I+K)^{-1}\|_{C^r[-1,1]\to C^r[-1,1]}<\infty$$

Now let n be any positive integer and X_n be the set of all algebraic polynomials of degree at most n (in $[-1,1]$). Define $I_n : C[-1,1]\to X_n$ to be the Legendre-Gauss interpolation operator:

$$(I_n w)(x)=\sum_{i=0}^{n}w_i L_i(x)$$

where $L_i(x)$ is the Legendre polynomial of degree i,

$$w_i=(i+1/2)\sum_{j=0}^{n}w(x_j)L_i(x_j)\omega_j,\quad \omega_j=\frac{2}{(1-x_j^2)(\partial_x L_{n+1}(x_j))^2}$$

and x_j are the $n+1$ roots of $L_{n+1}(x)$. And set $K_n : C[-1,1]\to C[-1,1]$ to be a discrete integral operator

$$(K_n w)(x)=\sum_{j=0}^{n}\omega_j k(x,x_j)w(x_j)$$