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INTERACTION OF THREE CONORMAL WAVES FOR SEMILINEAR WAVE EQUATIONS

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Abstract In this paper, we consider the interaction of triple of conormal waves with different singularities for semilinear wave equations. We will show that if three characteristic hyperplanes carrying different conormal singularities intersect transversally at the origin, then the solution will be conormal with respect to the three hyperplanes, and a new singularity will be produced on the surface of the light cone at later times. We can also prove here that the strength of the new singularity will be dependent only on the weakest one and strongest one in the three hyperplanes.

Key Words Semilinear wave equation; interaction of three singularities; conormal wave, second microlocalization.
Classification 35L.

1. Introduction

For a solution to a nonlinear system, when more than a pair of characteristic hypersufaces carrying conormal singularities intersect transversally in a lower dimensional manifold, a new singularity can form, even for a second order equation. An example shows that the new singularity can appear is due to J. Rauch and M. Reed (see [1]). The solution to a semilinear wave equation in two space dimensions is conormal in the past with respect to a triple of characteristic hyperplanes which intersect transversally at the origin, a new singularity is present at later times on the surface of the light cone over the origin. This result was established by J.M. Bony [2], M. Beals [3] and R. Melrose [4] individually. The general second order semilinear case was treated in J.Y. Chemin [5]. Recently, D.Y. Fang [6] considered the particular third order equations. But in those papers mentioned above, the authors only discuss the interaction of three waves with the same singularities.

In this paper, we shall study the interaction of three different singularities along three characteristic hyperplanes which intersect transversally at the origin. We will show that, in the future, the solution of semilinear wave equations has singularities not only on those hyperplanes, but also on the surface of the light cone over the origin at later times. And this singularity (on the light cone over the origin) satisfies the "sum

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law" for the weakest singularity and the strongest singularity. Our method is mainly based on the second microlocalization techniques (see [2]) and the conormal space with respect to different singularities (see [7]).

2. Notation and Results

Let u be a solution belonging to H^s for the semilinear wave equation

$$\Box u = f(t, x, y, u) \tag{2.1}$$

in an open neighborhood Ω of R^3 containing origin, where $\Box = \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}$, and f is a real C^∞ function. Put $\Omega_\pm = \Omega \cap \{\pm t > 0\}$, let Σ_1, Σ_2 and Σ_3 be characteristic hyperplanes for $\Box$, intersecting transversally at 0, and let Γ be the full light cone, $\Gamma^\pm$ the two half light cones in $\Omega_\pm$.

Let S be either a smooth hypersurface or the union of hypersurfaces intersecting only two by two and transversally, the space of conormal distribution $H^{s,k}(S)$ be the set of those $u \in H^s_{\text{loc}}(\Omega)$ such that $X^I u \in H^s_{\text{loc}}(\Omega)$ for $|I| \le k$, where X^I is the product of $|I|$ smooth vector fields tangent to Σ. Our main result is

Theorem 2.1 *If u is a solution of (2.1) belonging to $H^s_{\text{loc}}(\Omega)\left(s > \frac{3}{2} + 1\right)$, and assume that $u \in H^{s+l_j,m_j}(\Sigma_j)$ near Σ_j in Ω_- and $u \in H^{s+m}_{\text{loc}}(\Omega)$ away from $\Sigma_1 \cup \Sigma_2 \cup \Sigma_3$ in Ω_-, with $m = l_j + m_j$ $(j = 1, 2, 3)$, then for each $s' < s$, we have*

1) *$u \in H^{s'+m}_{\text{loc}}(\Omega)$ outside $\Sigma_1 \cup \Sigma_2 \cup \Sigma_3 \cup \Gamma^+$,*
2) *$u \in H^{s'+l_j,m_j}(\Sigma_j)$ near $\Sigma_j \Big\backslash \left(\bigcup_{i \ne j} \Sigma_i \cup \Gamma^+\right)$,*
3) *$u \in H^{s+\tau,m-\tau}(\Gamma^+)$ near $\Gamma^+ \backslash (\Sigma_1 \cup \Sigma_2 \cup \Sigma_3)$, with $\tau = \min\left(m, \left[s + \bar{l} + \underline{l} - \frac{3}{2} + 1\right]\right)$,*

$\bar{l} = \max(l_1, l_2, l_3)$, $\underline{l} = \min(l_1, l_2, l_3)$ ($[a]$ denote the integer part of a).

In this paper, we will use the second microlocalization techniques introduced by J.M. Bony. For convenience, we now recall some definitions and some results (see [2]) which will be used in this paper.

Definition 2.2 (2-microlocal Sobolev space) *Give $s \in R$, $k \in \mathbf{N}$, we shall say that $u = u(z_1, \cdots, z_n) \in N^{s,k}$, if*

$$z^\alpha D^\beta u \in H^s_{\text{loc}}(\Omega), \quad 0 \le |\alpha| = |\beta| \le k (k > 0)$$

and $u \in N^{s,-k}$, with $k > 0$, if and only if one has

$$u = \sum_{|\alpha| \le k} z^\alpha v_\alpha, \quad v_\alpha \in H^{s-|\alpha|}_{\text{loc}}$$

where $z \in R^n$, $z^\alpha = z_1^{\alpha_1} \cdots z_n^{\alpha_n}$, $D = (D_{z_1}, \cdots, D_{z_n})$.

Definition 2.3 (2-microlocal symbols)

$$a(z, \eta) \in \Sigma^{m,m'} \Longleftrightarrow |D^\alpha_\eta D^\beta_z a(z, \eta)| \le C_{\alpha\beta} \langle\eta\rangle^{m-|\alpha|+|\beta|} (1 + |z||\eta|)^{m'-|\beta|}$$