J.Partial Diff. Eqs., **7**(1994), 339–350
©International Academic Publishers Vol.7, No.4

ON INITIAL-BOUNDARY-VALUE PROBLEMS FOR A CLASS OF SYSTEMS OF QUASI-LINEAR EVOLUTION EQUATIONS

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(Received May 19, 1992; revised Sept. 21, 1992)

Abstract In this paper the initial-boundary-value problems for pseudo-hyperbolic system of quasi-linear equations:

$$\begin{cases} (-1)^M u_{tt} + A(x,t,U,V)u_{x^{2M}tt} = B(x,t,U,V)u_{x^{2M}t} + C(x,t,U,V)u_{x^{2M}} + f(x,t,U,V) \\ u_{x^k}(0,t) = \psi_{0k}(t), \quad u_{x^k}(l,t) = \psi_{1k}(t), \quad k = 0,1,\cdots,M-1 \\ u(x,0) = \varphi_0(x), \quad u_t(x,0) = \varphi_1(x) \end{cases}$$

is studied, where $U = (u, u_x, \cdots, u_{x^{2M-1}}), V = (u_t, u_{xt}, \cdots, u_{x^{2M-1}t}), A, B, C$ are $m \times m$ matrices, $u, f, \psi_{0k}, \psi_{1k}, \varphi_0, \varphi_1$ are m-dimensional vector functions. The existence and uniqueness of the generalized solution (in $H^2(0,T;H^{2M}(0,l))$) of the problems are proved.

Key Words Pseudo-hyperbolic system of quasi-linear equations of higher order.
Classification 35S.

1. Formulation of the Problems

In mathematical physics various problems for systems of quasi-linear differential equations of higher order are posed and considered, for example, see [1–2].

The problem to be considered in this paper is the initial-boundary-value problem for the system of quasi-linear pseudo-hyperbolic differential equations of higher order in the space $H^2(0,T;H^{2M}(0,l))$

$$\begin{cases} (-1)^M u_{tt} + A(x,t,U,V)u_{x^{2M}tt} = B(x,t,U,V)u_{x^{2M}t} \\ \quad +C(x,t,U,V)u_{x^{2M}} + f(x,t,U,V) & (1) \\ u_{x^k}(0,t) = \psi_{0k}(t), \quad u_{x^k}(l,t) = \psi_{1k}(t), \quad k = 0,1,\cdots,M-1 & (2) \\ u(x,0) = \varphi_0(x), \quad u_t(x,0) = \varphi_1(x) & (3) \end{cases}$$

where $A(x,t,U,V), B(x,t,U,V)$ and $C(x,t,U,V)$ are $m \times m$ matrices, $f(x,t,U,V)$ is a m-dimensional vector function, $\psi_{0k}(t), \psi_{1k}(t)$ $(k = 0,1,\cdots,M-1)$ and $\varphi_0(x), \varphi_1(x)$ are m-dimensional vector functions of $t \in [0,T]$ and $x \in [0,l]$ respectively. $U = (u, u_x, \cdots, u_{x^{2M-1}}), V = (u_t, u_{xt}, \cdots, u_{x^{2M-1}t})$.

2. The Initial-Boundary-Value Problems for Equations with Constant Coefficients

We consider the initial-boundary-value problems for

$$\begin{cases} (-1)^M u_{tt} + a_0 u_{x^{2M}tt} = f(x,t), \quad 0 < t \leq T, \qquad 0 < x < l & (4) \\ u_{x^k}(0,t) = \psi_{0k}(t), \qquad u_{x^k}(l,t) = \psi_{1k}(t), \quad k = 0,1,\cdots,M-1 & (5) \\ u(x,0) = \varphi_0(x), \qquad u_t(x,0) = \varphi_1(x), \quad a_0 = \text{const} > 0 & (6) \end{cases}$$

Theorem 1 *Let* $f(x,t) \in L_2(Q_T), \psi_{0k}(t), \psi_{1k}(t) \in H^2(0,T)$ *and* $\varphi_0(x), \varphi_1(x) \in H^{2M}(0,l)$ *such that*

$$\varphi_0^{(k)}(0) = \psi_{0k}(0), \quad \varphi_0^{(k)}(l) = \psi_{1k}(0) \quad \varphi_1^{(k)}(0) = \psi'_{0k}(0), \quad \varphi_1^{(k)}(l) = \psi'_{1k}(0), \\ k = 0,1,\cdots,M-1 \tag{7}$$

Then, the problems (4), (5) and (6) have precisely one generalized solution $u(x,t) \in H^2(0,T;H^{2M}(0,l))$ which satisfies the condition (4) in the generalized sense and the conditions (5), (6) in the classical sense. Furthermore, the following estimate holds

$$\|u\|_{H^2(0,T;H^{2M}(0,l))} \leq K_1\{\|f\|_{L^2(Q_T)} + \|\varphi_0\|_{H^{2M}(0,l)} + \|\varphi_1\|_{H^{2M}(0,l)} \\ + \sum_{k=0}^{M-1}(\|\psi_{0k}\|_{H^2(0,T)} + \|\psi_{1k}\|_{H^2(0,T)})\} K_1 = \text{const} \tag{8}$$

For the proof of Theorem 1 we first prove several lemmas.

Lemma 1 *Consider the boundary value problems for the equation*

$$\begin{cases} (-1)^M v + a_0 v_{x^{2M}} = f(x,t) & (9) \\ v_{x^k}(0,t) = \psi''_{0k}(t), \quad v_{x^k}(l,t) = \psi''_{1k}(t), \quad k = 0,1,\cdots,M-1 & (10) \end{cases}$$

where t is a parameter. Obviously, the problems (9), (10) possess a unique solution in the space $C^{2M,0}(Q_T)$, if $f(x,t) \in C(Q_T)$ and $\psi''_{0k}(t), \psi''_{1k}(t) \in C[0,T]$.

Lemma 2 *Consider the initial value problem for ordinary differential equation*

$$u_{tt} = v(x,t) \tag{11}$$

with initial condition (6), where $x \in [0,l]$ is a parameter, $v(x,t)$ is a unique solution of problems (9), (10) in the space $C^{2M,0}(Q_T)$.

If $\varphi_0(x), \varphi_1(x) \in C^{2M,0}[0,l]$ satisfy the condition (7), for problem (11), (6) there exists a unique solution in the space $C^{2M,0}(Q_T)$.

Proof

$$u(x,t) = \varphi_0(x) + \varphi_1(x)t + \int_0^t \int_0^\tau v(x,y)dyd\tau \tag{12}$$