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THE GLOBAL ATTRACTORS FOR THE PERIODIC INITIAL VALUE PROBLEM OF GENERALIZED KURAMOTO-SIVASHINSKY TYPE EQUATIONS IN MULTI-DIMENSIONS

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Dedicated to the 70th birthday of Professor Zhou Yulin

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Abstract The existence of global attractors for the periodic initial value problem of generalized Kuramoto-Sivashinsky type equations in multi-dimensions is proved. We also get the estimates of the upper bounds of Hausdorff and fractal dimensions for the global attractors by means of a uniform priori estimates for time.

Key Words The global attractors; Hausdorff dimensions; fractal dimensions.
Classification 35Q.

1. Introduction

The equation

$$\varphi_t + \frac{1}{2}(\nabla\varphi)^2 + \nu\varphi + \alpha\Delta\varphi + \gamma\Delta^2\varphi = 0 \tag{1.0}$$

was independently advocated by Kuramoto [1] (1978), in connection with reaction-diffusion systems, and Sivashinsky [2] (1977), in modeling flame propagation, where α, γ and ν are positive constants. It also arises in the context of viscous film flow [3] (1982), and the bifurcating solutions of the Navier-Stokes equations [4]. In [5–8] the global attractors and the bifurcating solutions for the equation (1.0) ($n = 1$) with periodic boundary conditions have been studied. In [9] B.Nicolaenko has proposed a generalized KS type equation. In [10–12], [15] and [16], the existence of global smooth solution, the asymtotic behavior as $t \to \infty$, the structure of travelling wave solution, the similarity solutions obtained by using the methods of Lie's group and infinitesimal transformation, and the numerical solutions by spectral method for the generalized KS type equation and system have been investigated. In [17], the existence of global attractors for the periodic initial value problem of generalized Kuramoto-Sivashinsky type equations in one dimension is proved, and the estimates of the upper bounds of Hausdorff and fractal dimensions are given.

In this paper we study the global attractors for the following periodic initial value problem of a class of global KS type equations in multi-dimensions

$$u_t + \alpha\Delta u + \gamma\Delta^2 u + \nabla \cdot f(u) + \Delta\varphi(u) = g(u) + h(x) \tag{1.1}$$

$$u(x,t) = u(x+2de_i,t), \quad x \in \Omega, t \geq 0, j = 1, \cdots, n \tag{1.2}$$

$$u(x,t)|_{t=0} = u_0(x), \quad x \in \Omega \tag{1.3}$$

where $\Omega \subset R^n$ is an n-dimensional cube with width $2d$ in each direction, that is

$$\overline{\Omega} = \{x = (x_1, \cdots, x_n)|_{|x_i| \leq d}, \quad i = 1, \cdots, n\}$$

where $x + 2de_i$ denotes $(x_1, \cdots, x_{i-1}, x_i + 2d, x_{i+1}, \cdots, x_n)$ $(i = 1, \cdots, n)$, the constants $\alpha \geq 0$ and $\gamma > 0, \nabla \cdot f(u) \equiv \sum_{k=1}^{n} \frac{\partial f_k(u)}{\partial x_k}, \Delta u = \sum_{k=1}^{n} \frac{\partial^2 u}{\partial x_k^2}$. It is well known that as $\alpha = 0$, $f(u) = 0$ in (1.1), the equation is called the Cahn-Hilliard equation with nonhomogeneous term. In Section 2 we establish the t-independent a priori estimates of problem (1.1), (1.2), (1.3). In Section 3 the existence and uniqueness of global smooth solution for problem (1.1) (1.2) (1.3) are proved. In Section 4 we prove the existence of global attractor of problem (1.1) (1.2) (1.3), and get the estimates of upper bounds of Hausdorff and fractal dimensions for the global attractors.

[Note] In this paper, we denote the norm $\|\cdot\|_{L_2(\Omega)}$ by $\|\cdot\|$, and $\|\cdot\|_{L_\infty(\Omega)}$ by $\|\cdot\|_\infty$.

2. t-independent *a Priori* Estimates of Problem (1.1)–(1.3)

Lemma 1 *Suppose that*

(1) $\varphi'(u) \leq \varphi_0$,

(2) $\gamma > \dfrac{\alpha + \varphi_0}{2}$

(3) $g(0) = 0, g'(u) \leq g_0, g_0 < -\dfrac{(\alpha + \varphi_0 + 1)}{2}$,

(4) $h(x) \in L_2(\Omega), u_0(x) \in L_2(\Omega)$.

Then, for the smooth solution of problem (1.1)–(1.3), we have the following estimate

$$\|u(\cdot,t)\|^2 \leq e^{(2g_0+\alpha+1+\varphi_0)t}\|u_0(\cdot)\|^2 + \frac{1}{|2g_0+\alpha+\varphi_0+1|}(1 - e^{(2g_0+\alpha+1+\varphi_0)t})\|h(x)\|^2 (0 \leq t < \infty) \tag{2.1}$$

Furthermore, we have

$$\limsup_{t\to\infty} \|u(\cdot,t)\|^2 \leq \frac{\|h(x)\|^2}{|2g_0+\alpha+\varphi_0+1|} = E_0$$

$$\limsup_{t\to\infty} \frac{1}{t}\int_0^t \|\Delta u(\cdot,t)\|^2 dt \leq \frac{1}{[2\gamma - (\alpha+\varphi_0)]}[\|u_0(x)\|^2 + \|h(x)\|^2] \tag{2.2}$$

Proof Take the inner product of (1.1) with u, we have

$$(u, u_t + \alpha\Delta u + \gamma\Delta^2 u + \nabla \cdot f(u) + \Delta\varphi(u) - g(u) - h(x)) = 0 \tag{2.3}$$