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ON THE SEMICONDUCTOR SYSTEM

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Abstract In this paper, the semiconductor system is discussed. The existence and uniqueness of the global solution of the carrier transport problem are obtained. Under the condition that the width in some direction of the domain being sufficiently small, the existence and uniqueness of the solution of the steady states are proved. It is also proved that the solution of the carrier transport problem tends to the solution of the steady states problem exponentially when t goes to infinity.

Key Words Semiconductor; strongly coupled system; upper-lower solution method; carrier transport; steady state; existence; uniqueness; asymptotic behaviour
Classification 35K57

Semiconductor system[1] is a semilinear partial differential system as follows

$$\begin{cases} \dfrac{1}{\mu_n} n_t = \Delta n - \nabla\cdot(n\nabla\phi) - R(n,p) \\ \dfrac{1}{\mu_p} p_t = \Delta p + \nabla\cdot(p\nabla\phi) - S(n,p) \qquad \text{in } Q_T = \Omega\times[0,T] \\ \Delta\phi = 4\pi\eta(n - p - N_Y(x,t)) \end{cases} \tag{0.1}$$

The initial and boundary conditions are

$$\begin{cases} n|_{\partial\Omega} = \bar{n}(x,t), \quad p|_{\partial\Omega} = \bar{p}(x,t), \quad \phi|_{\partial\Omega} = \bar{\phi}(x,t) & \text{on } \partial\Omega\times[0,T] \\ n(x,0) = n_0(x), \quad p(x,0) = p_0(x) & \text{on } \bar{\Omega} \end{cases} \tag{0.2}$$

where $x = (x_1, x_2, \cdots, x_N), \Omega \subset R^N, N \geq 1; n, p$ are the densities of mobile holes and electrons respectively, ϕ is the electrostatic potential; $R(n,p) = r(n,p)(np-1)$, $S(n,p) = s(n,p)(np-1)$, $r(n,p)$ and $s(n,p)$ are positive Lip-continuous functions, $N_Y(x,t)$ is a positive smoothing function, η is a positive constant and

$$0 \leq r \leq \bar{r}, \quad 0 \leq s \leq \bar{s}, \quad 0 < N_Y < \bar{N}_Y \tag{0.3}$$

$$0 \leq \bar{n}, \bar{p}, n_0, p_0 \leq 1 \tag{0.4}$$

$$\bar{n},\bar{p},\bar{\phi}\in C^{2+\alpha,1+\frac{\alpha}{2}}(Q_T),\quad n_0,p_0\in C^{2+\alpha}(\Omega) \tag{0.5}$$

$$|\bar{n}-\bar{n}_\infty(x)|,\ |\bar{p}-\bar{p}_\infty(x)|,\ |\bar{\phi}-\bar{\phi}_\infty(x)|\leq Ce^{-\gamma t},\quad \gamma>0 \tag{0.6}$$

These functions also satisfy the conditions of compatibility.

The steady states of this problem are defined as follows

$$\begin{cases} -\Delta n=-\nabla\cdot(n\nabla\phi)-R(n,p) \\ -\Delta p=\nabla\cdot(p\nabla\phi)-S(n,p) \qquad \text{in } \Omega \\ \Delta\phi=4\pi\eta(n-p-N_Y) \end{cases} \tag{0.7}$$

$$n|_{\partial\Omega}=\bar{n}_\infty,\quad p|_{\partial\Omega}=\bar{p}_\infty,\quad \phi|_{\partial\Omega}=\bar{\phi}_\infty \tag{0.8}$$

They are strong coupled systems. Many people are interested in these two problems. There have been many works about them already in various ways and under different conditions, see [2]–[5].

In this paper, we use the upper-lower solution method[6],[7] to discuss these problems. We have the following results:

1. The existence and uniqueness of the global solution of the carrier transport problem (0.1), (0.2) under the conditions (0.3)–(0.5) are obtained. The solution is bounded and positive.

2. The existence and uniqueness of the solution of the steady states problem (0.7)–(0.8) under the condition of the domain Ω being sufficiently small in one direction are obtained. This condition means the matter of semiconductor is thin in one direction physically. In fact, P-N junction in semiconductor is very thin. The solution we get is also bounded and positive.

3. Under the condition of the domain Ω being sufficiently small in one direction and (0.6), the solution of the carrier transport problem (0.1)–(0.2) tends to the solution of the steady states problem (0.7)–(0.8) when t goes to infinity exponentially.

1. The Carrier Transport Problem

Consider a space

$$\mathcal{H}=L_\infty(0,T;L_q(\Omega)),\quad T>0,\quad 1<q<\infty$$

and a set

$$\mathcal{E}=\{(u,v)\in\mathcal{H}|0\leq(u,v)\leq e^{Mt}\}$$

where M will be determined later; $(a,b)\ \leq,(\geq)C$ means $a\ \leq,(\geq)C$ and $b\ \leq,(\geq)C$; $(a,b)\leq(c,d)$ means $a\leq c, b\leq d$.