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FORMATION OF SINGULARITIES OF SOLUTIONS FOR CAUCHY PROBLEM OF QUASILINEAR HYPERBOLIC SYSTEMS WITH DISSIPATIVE TERMS[1]

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Abstract In this paper, for a class of 2×2 quasilinear hyperbolic systems, we get existence theorems of the global smooth solutions of its Cauchy problem, under a certain hypotheses. In addition, for two concrete quasilinear hyperbolic systems, we study the formation of the singularities of the C^1-solution to its Cauchy problem.

Key Words Quasilinear hyperbolic systems; Cauchy problem; global smooth solution; singularity.

Classification 35L65.

1. Introduction

For the first order quasilinear hyperbolic systems

$$\begin{aligned} &u_t + \sigma(v)_x + 2\alpha u = 0, \quad \alpha > 0 \\ &v_t - u_x = 0 \end{aligned} \tag{1.1}$$

the existence and nonexistence of global smooth solutions of its Cauchy problem or initial-boundary problem had been studied by many scholars (see [1, 2, 3, 4, 5])

Suppose that there exists a constant $R > 0$, such that

$$\sigma'(v) < 0, \quad \forall |v| < R, \quad \sigma(v) \in C^2(|v| < R) \tag{1.2}$$

For (1.1), the initial datum are given by

$$u(x,0) = u_0(x), \qquad v(x,0) = v_0(x) \tag{1.3}$$

Let z, w be the Riemann invariants, i.e.,

$$z = u + \varphi(v), \qquad w = u - \varphi(v) \tag{1.4}$$

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where $\varphi(v) = \int_0^v \sqrt{-\sigma'(v)}dv$. Suppose that $z_0(x)$, $w_0(x)$, which are determined by the initial datum $u_0(x), v_0(x)$, satisfy

$$z_0(x), w_0(x) \in C^1(R)$$

Nishida had proved that Cauchy problem (1.1), (1.3) admits a global smooth solution, if the C^1-norm of $w_0(x)$, $z_0(x)$ is sufficiently small (see [1]).

Under the hypothesis of "smallness" to the C^1-norm of the initial datum, Li Tatsiens' have spread Nishida's result of existence into Cauchy problem of the general $n \times n$ systems with dissipative terms of the diagonal dominant ([6, 7, 8]):

$$U_t + F(U)_x + G(U) = 0 \tag{1.5}$$

However, for the nonlinear vector-function $G(U)$ and weakly diagonal dominant $A = B(0)\nabla G(0)B^{-1}(0)$, whether Cauchy problem to (1.5) admits a global smooth solution, Li Tatsiens' have not been studying this case, where $B(U)$ is the matrix of the eigenvector to (1.5), and det $B(U) \neq 0, B^{-1}(U)$ is a inverse matrix of $B(U)$. In this paper, for the case of that, we show that Cauchy problem of systems does not admit a global smooth solution, even if the smallness of C^1-norm or C^0-norm of the initial datum is ensured.

Under the hypothesis of the monotonic initial datum, Li Caizhongs' have shown that Cauchy problem of (1.1) admits a global smooth solution, if the oscillation of the initial datum is small, and Cauchy problem of (1.1) has a C^1-solution for only a finite time ([5]).

In view of the weakly diagonal dominant in (1.1), we compare Li Tatsiens' result of the strictly diagonal dominant, a query is easily arised: whether the singular result can be avoided in [5], if the dissipative terms are strengthened. In this paper, we show that the singular result is not yet avoided, even if the dissipative terms are strengthened.

In addition, we spread the existence result gotten by Li Caizhongs' to Cauchy problem of the general 2×2 systems:

$$\begin{aligned} u_t + \lambda(u,v)u_x + f(u,v) = 0 \\ v_t + \mu(u,v)v_x + g(u,v) = 0 \end{aligned} \tag{1.6}$$

2. The Existence Theorems of the Global Smooth Solutions

Consider Cauchy problem of the quasilinear equations as

$$\begin{aligned} u_t + \lambda(u,v)u_x + f(u,v) = 0 \\ v_t + \mu(u,v)v_x + g(u,v) = 0 \end{aligned} \tag{2.1}$$