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THE SOLVABILITY FOR A CLASS OF NONLINEAR INTEGRODIFFERENTIAL EQUATIONS OF PARABOLIC TYPE[1]

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Abstract In this paper we consider the well-posedness for a class of nonlinear integrodifferential equations of parabolic type. We use integral estimates to deduce an *a priori* estimate in the classical space $C^{2+\alpha,1+\frac{\alpha}{2}}$. The existence of the solution is established by means of the continuity method which is similar to a parabolic initial and boundary value problem. Moreover, the continuous dependence upon the data and the uniqueness of the solution are obtained. Finally, the results are generalized into a class of nonlinear integrodifferential systems.
Key Words Integrodifferential equation; nonlinearity; global solvability.
Classifications 35A05, 45K05.

1. Introduction

Consider the following integrodifferential equation of parabolic type

$$\frac{\partial u}{\partial t}=\frac{\partial}{\partial x}\left[a(x,t,u)\frac{\partial u}{\partial x}\right]+\int_0^t\left\{\frac{\partial}{\partial x}[b(x,t,u,u_x)]+c(x,t,u,u_x)\right\}dt,\quad (x,t)\in Q_T \quad (1.1)$$

with the initial and boundary conditions

$$u(0,t)=f_1(t),\qquad 0\le t\le T \quad (1.2)$$

$$u(1,t)=f_2(t),\qquad 0\le t\le T \quad (1.3)$$

$$u(x,0)=u_0(x),\qquad 0\le x\le 1 \quad (1.4)$$

where $Q_T=\{(x,t):0<x<1,0<t<T\}$.

The problem (1.1)–(1.4) can be used as various mathematical models for many physical processes in the fields of heat transfer and thermoelasticity in which the effect of past history is taken into account. The reader can find these models in [24] and the

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references over there. On the other hand, if one takes the derivative with respect to the variable t to Equation (1.1), he obtains a mixed type of equation which describes the propagation of disturbance in viscous media (cf. [2], [8], [18], [20], etc.). Motivated by these physical models, we are interested in the existence and uniqueness of the solution for the problem (1.1)–(1.4). The study for such kind of integrodifferential equations has been developed for many years. A large number of authors (cf. e.g. [1], [6], [7], [12], [16], [19], [21]) write the equation into the following abstract form

$$\frac{du(t)}{dt} + A(t, u(t))u(t) = g(t, u(t), \int_0^t h(x, s, u(s))ds)$$

in a Banach space. Under the various assumptions on the operator A and the function g, they employ the semigroup theory to establish the existence as well as the uniqueness of the solution. Moreover, several people investigate the problem from another point of view. They treat Equation (1.1) as a mixed type one with the proper initial and boundary conditions (Refer to [2], [5], [8], [26], [28], e.g.). Some of them apply the argument of separation variables to express the solution as a series and obtain the existence, uniqueness and the asymptotic behavior of the solution. The authors of [20] consider the problem in n-dimensional space and obtain the global solvability *via* the method of continuity. Other approaches such as the contracting mapping principle, compactness, etc. are also applied to establish the existence and the uniqueness of the solution. However, most of these previous works on the global solvability deal with the equation in which the principal part is linear even in one space dimension. Recently, the author of [31] considers the problem from a rather different point of view. He uses the argument which is developed in [32] to obtain the global solvability for a class of nonlinear integrodifferential equations. In this paper, we follow the idea of [31] and deal with a different class of nonlinear integrodifferential equations. The classical solution for the problem (1.1)–(1.4) is established by means of the classical continuity method similar to a parabolic initial and boundary value problem. It is worth while pointing out that our method is also valid for a class of quasilinear integrodifferential systems.

Throughout this paper, without loss of generality we assume that

$$f_1(t) = f_2(t) = 0$$

We also assume the following conditions hold.

H(1) $a(x, t, u) \in C^{3,3,2}(\bar{Q}_T \times R^1)$ and $a(x, t, u) \geq a_0 > 0$ for $(x, t, u) \in \bar{Q}_T \times R^1$.

H(2) $b(x, t, u, p), c(x, t, u, p) \in C^{2+\alpha,1+\alpha,2,2}(\bar{Q}_T \times R^2)$. Moreover,

(a) $|b(x, t, u, p)| + |c(x, t, u, p)| \leq M_0[1 + |u| + |p|], (x, t, u, p) \in \bar{Q}_T \times R^2$,

(b) $|b_x(x, t, u, p)| \leq M_2(|u|)[1 + |p|]$ and $|b_u(x, t, u, p)| + |b_p(x, t, u, p)| \leq M_2(|u|)$ for $p \in R^1$,

where M_0 is a constant, $M_1(s)$ and $M_2(s)$ are two known increasing functions of s.