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THE ASYMPTOTIC EXPANSION OF SOLUTIONS FOR THE NAVIER-STOKES EQUATIONS WITH LARGE PARAMETERS

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Abstract In this paper, we decomposite the coefficient matrix of the Navier-Stokes equations with large parameter in λ power. By factor analysis and energy estimation, the long time existence and asymptotic expansion of the solution for the system are obtained.

Key words Navier-Stokes equations; expansion; approximation.
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1. Introduction

The partial differential system with large parameter in the coefficient are interesting, because it has wide variety physical meaning[1]-[5].

In this paper, we study the Cauchy problem of compressible fluid Navier-Stokes equations with large parameter. The system and initial conditions may be written as follows

$$\frac{\partial\rho^\lambda}{\partial t}+\operatorname{div}(\rho^\lambda v^\lambda)=0 \tag{1.1}$$

$$\rho^\lambda\left(\frac{\partial v^\lambda}{\partial t}+(v^\lambda\cdot\nabla)v^\lambda+\lambda^2\nabla p^\lambda(\rho)\right)=(\mu+\nu)\nabla(\operatorname{div}v^\lambda)+\nu\Delta v^\lambda \tag{1.2}$$

$$(\rho^\lambda(x,0),v^\lambda(x,0))=(\rho_0(x,\lambda),v_0(x,\lambda)) \tag{1.3}$$

where ρ^λ is the fluid density, $v^\lambda=(v_1^\lambda,v_2^\lambda,v_3^\lambda)$ are it's velocity, p^λ is pressure, $p=p(\rho)$ is a given equation of state, $\frac{dp}{d\rho}>0(\rho>0)$, ∇ is gradient operator, div is divergent operator, Δ is Laplace operator. The definition of Mach number of fluid motion is $M=|v_m|\left(\frac{dp(\rho_m)}{d\rho}\right)^{-1/2}$, where v_m,ρ_m are mean velocity and mean density. The parameter $\lambda=C_0M^{-1}$ is essentially the inverse of the initial Mach number, while it tends to large, the problem is called the large parameter problem, μ,ν are viscous coefficients of fluid flow, they are the functions of ρ and $0\leqslant\nu\leqslant\nu_0$. The following relations are established

$$\frac{1}{C}\nu\leqslant\mu+2\nu\leqslant C\nu$$

where C_0,ν_0,C are positive constants.

In [3], the limit state of Cauchy problem (1. 1)—(1. 3) are already studied, when the λ, γ arise singular. Under certain conditions, we proved that as the $\lambda \to \infty$, the solutions of (1. 1)—(1. 3), $\rho^\lambda \to \rho_0$ (some positive constant), $v^\lambda \to v^\infty$, and v^∞ is the solution of Cauchy problem of incompressible flow Navier-Stokes equations

$$\operatorname{div}(v^\infty) = 0 \tag{1.4}$$

$$\rho_0\left(\frac{\partial v^\infty}{\partial t} + (v^\infty \cdot \nabla)v^\infty\right) = -\nabla p^\infty + \gamma \Delta v^\infty \tag{1.5}$$

$$v^\infty(x,0) = v_0(x) \tag{1.6}$$

The problem to be studied in this paper is that the solution of the system (1. 1)—(1. 3) can be gradually developed with the negative power of λ, and the long existence of the solution. The method used in this paper is to decompose the coefficient matrix and right hand of (1. 1)—(1. 2) according to the power order of λ, then make an energy estimate of the concerned having been taken out after making factor analysis, thus obtain the desired results by using the iterative principle.

For the convenience of describing the problem, we write the right hand of (1. 2) as

$$D(\gamma)v = (\mu + \gamma)\nabla(\operatorname{div} v) + \gamma \Delta v$$

It is obvious

$$\int_{R^N} (D(\gamma)v, v)dx \leqslant -C_1\gamma \| \nabla v \|^2 \tag{1.7}$$

$$\| D(\gamma)v \|^2 \leqslant C_2\gamma^2 \| \nabla v \|_1^2 . \tag{1.8}$$

where C_1, C_2 are constants, $(\cdot, \cdot)$ is inner product on $\mathbf{R}^N$, the spatial variate $x \in R^N$. $H^s(\mathbf{R}^N)$ denotes the s-th Sobolev space on $\mathbf{R}^N$, its norm is written as $\| \cdot \|_s$, $\| \cdot \|_0$ is the L^2 norm $\| \cdot \|$. Let $s_0 = \left[\frac{N}{2}\right] + 1$, $s \geqslant s_0 + 2$ be a positive integer. If function $u(x,t) \in L^\infty(0,T;H^s(\mathbf{R}^N))$, its norm is definded as

$$[u]_{s,T} = [u]_s = \max_{0 \leqslant t \leqslant T} \| u(t) \|_s \tag{1.9}$$

2. Main Results

To meet the requirement of proof we write the pressure p^λ as unknown function, so that

$$\left(\rho^\lambda \frac{dp^\lambda}{d\rho}\right)^{-1}\left(\frac{\partial p^\lambda}{\partial t} + v^\lambda \cdot \nabla p^\lambda\right) + \operatorname{div} v^\lambda = 0 \tag{2.1}$$