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THE FOUNDAMENTAL SOLUTION OF WEIGHTED CAUCHY PROBLEM FOR ONE-ORDERED FUCHSIAN TYPE PSEUDODIFFERENTIAL OPERATOR①

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Abstract

In this paper, we begin with introducing two classes of generalized pseudo-homogeneous functions θ^m and T^m, studying their basic properties. Then, under the most general hypothesis, we get the global foundamental solution of weighted Cauchy problem for one-order Fuchsian-type pseudodifferential operator in the frame PsDO whose symbol belongs to $C(D, S^m_{1,0})$.

Introduction

Early in 1973, M. S. Baouendi and C. Goulaouic studied Fuchsian-type partial differential equation. They had considered classical Cauchy problem of the operator:

$$P(t, x, D_t, D_x) = t^k D_t^m + a_{m-1}(x) t^{k-1} D_t^{m-1} + \dots + a_{m-k}(x) D_t^{m-k} + \dots + \sum_{p<m} \sum_{|\beta| \le m-p} t^{\alpha(p,\beta)} D_t^p a_{p,\beta}(t, x) D_x^\beta \qquad (0.1)$$

where $\alpha(p, \beta) = \max\ (0,\ k + p - m + 1)$. Under a given hypothesis, they had proved the existence and uniqueness of the solution in the frame of analytic functions. Qi Minyou [3, 4] had popularized (0.1) to the Fuchsian-type equations whose coefficients are of operator and to higher dimensional, singular manifolds. He had proved the existence of Nilsson solution and non-Nilsson solution. At the same time, Qiu Qingjiu [2] studied the parametrix of Cauchy problem for the operator $t\partial_t + B(t, x, D_x)$, obtained the parametrix of Cauchy problem with initial condition given by $t = s > 0$, while he assumed that B is m (>1)-order elliptic PsDO for x. Just as [2] said that it was not Fuchsian-type any more. Later one in [5] studied the local solvability to a sort of full

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characteristic operator. But it's not as the model as in[1]. Therefore, to Fuchsian-type PDE studied in[1], how do we put their Cauchy problemin C^∞ frame? Whether are they globally solvable? This is a problem which has not been solved so far.

In this article, we have studied the global solvability to the weighted Cauchy problem of the one-order Fuchsian-type PsDO as following:

$$\begin{aligned} Pu = [t\partial_t - \lambda(x) + tA(t, x, D_x)]u = t^{\lambda(x)+1} f(t, x) \\ t^{-\lambda(x)} u|_{t\to 0} = \varphi(x) \in \mathscr{E}'(R^n) \end{aligned} \qquad (0.2)$$

where $f \in t^{-\frac{1}{2}} C(\overline{R}_+, \mathscr{E}'(R^n))$, $A(t, x, D_x)u = \int e^{ix\cdot\xi} a(t, x, \xi)\tilde{u}(t, \xi) \bar{d}\xi$, $a(t, x, \xi) \in S^1_{\mathrm{phg}}$,

i. e. $a(t, x, \xi) \sim \sum_{j=0}^{\infty} a_j(t, x, \xi)$; $a_j(t, x, \xi)$ is positive-homogeneous with respect to ξ of degree $1-j$, C^∞ function with respect to (t, x) and bounded to x in C^∞.

This paper's arrangement: In Section 1, we introduce the functional classes θ^m and T^m, then study their basic properties; In 2, we constructure the parametrix; In 3, we constructure the foundamental solution of (0.2), from this, we obtain the expression of global solution.

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For simplicity, we denote $x = (x_1, x_2, ..., x_n)$, the Fourier transformation of $u(t, x)$ with respect to x by $\tilde{u}(t, \xi)$, $\overline{R}_+ = \{t;\ t \geq 0\}$, the tempered distribution space by $\mathscr{S}'(R^n)$. Suppose the condition (H) for (0.2) holds:

(H_1): $a_0(t, x, \xi) \geq C(T)|\xi|$, $(C(T) \geq 0)$, for $\forall$ $(t, x, \xi) \in [0, T] \times R^n \times R^n/0$

(H_2): $\lambda(x)$ is C^∞-bounded founction with respect to x.

First of all, we define a class of generalized pseudohomogeneous functions as following:

Definition 1 $\theta^m = \{f(t, \tau, x, \xi) \mid f = \sum_{i\in I} g_i(t, \tau, \ln t, \ln\tau) h_i(\tau, x, \xi) e^{-(t-\tau)a_0(\tau, x, \xi)}$ *here g_i are polynomial functions, $h_i \in C^\infty(\overline{R}_+ \times R^n \times R^n/0)$ and for an arbitrary $\delta > 0$, $\forall$ $(t, \tau, x, \xi) \in R_+ \times R_+ \times R^n \times R^n/0$, there always is $g_i(\delta^{-1}t, \delta^{-1}\tau, \ln t, \ln\tau) h_i(\tau, x, \delta\xi) = \delta^m g_i(t, \tau, \ln t, \ln\tau) h_i(\tau, x, \xi)$, where I is finite set$\}$.*

By virture of Definition 1, one can easily prove

Proposition 1 *Suppose $f \in \theta^m$, then*

1° $D_x^\alpha f \in \theta^m$, $\forall$ $\alpha \in Z_+^n$ *(n-multiplicate positive integer set)*

2° $t\partial_t f \in \theta^m$, $e^{-(t-\tau)a_0} \int_\tau^t e^{(s-\tau)a_0} f(s, \tau, x, \xi) ds \in \theta^m$.

Definition 2 $T^m = \bigcup_{\alpha_1 \leq m} \Pi^{\alpha_1}$, *$\alpha_1$ is finite set, where* $\Pi^{\alpha_1} = \{f(t, \tau, x, \xi) \mid$