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Immersed Interface CIP for One Dimensional Hyperbolic Equations

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Abstract. The immersed interface technique is incorporated into CIP method to solve one-dimensional hyperbolic equations with piecewise constant coefficients. The proposed method achieves the third order of accuracy in time and space in the vicinity of the interface where the coefficients have jump discontinuities, which is the same order of accuracy of the standard CIP scheme. Some numerical tests are given to verify the accuracy of the proposed method.

AMS subject classifications: 65M12, 65M25

Key words: CIP method, immersed interface method, hyperbolic equations in discontinuous media.

1 Introduction

In this paper we develop a higher order numerical method for one dimensional hyperbolic equations with discontinuous coefficients, for instance, the scaler advection equation

$$u_t+(c(x)u)_x=0, \quad t>0,\ x\in\mathbb{R}, \tag{1.1}$$

and one dimensional Maxwell's equations

$$\varepsilon(x)E_t=H_x, \quad \mu(x)H_t=E_x, \quad t>0, \ x\in\mathbb{R}. \tag{1.2}$$

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The building blocks of the method consist of CIP method and Immersed interface method.

CIP method proposed in [10, 13, 14] approximates the solution to a differential equation through the evolution of more than one degree of freedom per cell, e.g., not only the primitive variable but also its derivatives or other quantities such as the cell average of the solution. The method uses the exact integration for the constant transport equation (the method of characteristics for the constant wave speed) and the time splitting methods for the space varying perturbations, and achieves a higher order of accuracy using a compact stencil. The CIP method has been widely used as an accurate numerical solver for differential equations, which is less-dispersive and less-dissipative than some of the other methods. Here is an incomplete list of references for CIP and related works: nonlinear hyperbolic equations [13], the multi-phase analysis [16], a multi-dimensional the Maxwell's equations [8], light propagation in dielectric media [3], a new mesh system applicable to non-orthogonal coordinate system [15], a numerical investigation of the stability and the accuracy [11]. For the other methods closely related to CIP, we refer to [2, 5].

If the coefficient $c(x)$ has a jump discontinuity, the CIP method without extra care of the discontinuity leads to loss of accuracy. In order to maintain the accuracy in the numerical solution in a neighborhood of the interface, a special treatment should be made. Immersed interface method is one of the technique for the interface problems, which was developed in [6, 17]: A standard numerical method is modified locally near the interface according to the interface relations so that the accuracy in the numerical solution is maintained in the entire domain. For more information, see the monograph [7].

In this article, we develop an immersed interface method for CIP that provides third order accuracy at all grid points, including the adjacent grid to the interface. We construct, by using proper interface conditions, a piecewise Hermite type interpolation on a cell that contains a point of jump discontinuity in $c(x)$.

An outline of our presentation is as follows: Section 2 gives a brief introduction on CIP methods proposed in [13, 14]. Section 3, the main part of the article, is concerned with developing a CIP scheme for discontinuous media. In Section 4 its application to the Maxwell's system is presented. Sections 3 and 4 include some numerical tests to verify the accuracy of the proposed method.

2 Review of CIP

We give a brief review of a CIP method for Eq. (1.1) as a model equation. We first treat the case where the velocity c is a positive constant, and move on to the case of c being a variable velocity.

2.1 CIP for constant velocity

Let us denote the derivative $u_x(t,x)$ by $v(t,x)$. For the constant velocity c, the equations for u and v are