Commun. Comput. Phys.
doi: 10.4208/cicp.171011.180712a

Vol. **14**, No. 1, pp. 126-152
July 2013

A Lattice Boltzmann and Immersed Boundary Scheme for Model Blood Flow in Constricted Pipes: Part 1 – Steady Flow

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Received 17 October 2011; Accepted (in revised version) 18 July 2012

Communicated by Kazuo Aoki

Available online 30 October 2012

Abstract. Hemodynamics is a complex problem with several distinct characteristics; fluid is non-Newtonian, flow is pulsatile in nature, flow is three-dimensional due to cholesterol/plague built up, and blood vessel wall is elastic. In order to simulate this type of flows accurately, any proposed numerical scheme has to be able to replicate these characteristics correctly, efficiently, as well as individually and collectively. Since the equations of the finite difference lattice Boltzmann method (FDLBM) are hyperbolic, and can be solved using Cartesian grids locally, explicitly and efficiently on parallel computers, a program of study to develop a viable FDLBM numerical scheme that can mimic these characteristics individually in any model blood flow problem was initiated. The present objective is to first develop a steady FDLBM with an immersed boundary (IB) method to model blood flow in stenoic artery over a range of Reynolds numbers. The resulting equations in the FDLBM/IB numerical scheme can still be solved using Cartesian grids; thus, changing complex artery geometry can be treated without resorting to grid generation. The FDLBM/IB numerical scheme is validated against known data and is then used to study Newtonian and non-Newtonian fluid flow through constricted tubes. The investigation aims to gain insight into the constricted flow behavior and the non-Newtonian fluid effect on this behavior.

AMS subject classifications: 76Z05, 76M20, 65N06

Key words: Finite difference method, lattice Boltzmann method, immersed boundary method, blood flow, constricted pipe.

1 Introduction

Flow behavior in a stenosed artery is quite different from that in normal ones. Viscous stresses and resistance to flow are much higher in stenosed arteries compared to their

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normal counterparts. If the stenosis is severe enough, high blood flow speeding through narrow stenosis will give rise to considerable compressive stress in the tube wall, and critical flow conditions such as negative pressure, high shear stress and flow separation will develop. These flow and mechanical conditions can cause artery fatigue, arterial compression flow limitation, and possible flow-induced instabilities. The resulting static and dynamic loading on the diseased arterial wall may be sufficiently vigorous or sustained to fracture the plaque cap of the stenosis, thus causing fragments to sweep downstream. This process could directly lead to serious clinical consequences, such as strokes or heart attacks [1–3]. In view of this rather complicated and complex flow behavior, any numerical scheme used to investigate hemodynamics should at least be able to handle pulsating flow of Newtonian and non-Newtonian fluids through three-dimensional (3-D) tubes with and without constrictions where the boundary geometry changes with time, and with solid and/or elastic boundaries.

Lattice Boltzmann method (LBM) has developed into an alternative powerful numerical solver for the Navier-Stokes (NS) equations and for modeling flow physics [4–6]. The equation is hyperbolic and can be solved locally, explicitly, and efficiently on parallel computers [7, 8]. Consequently, the applicability and suitability of LBM to the field of blood flow has been widely explored by researchers [7–14]. Real arterial stenoses rarely have well-defined geometric shapes and stenotic plaques usually have complex geometries. Therefore, it is difficult to apply the boundary conforming grid method to such problems. Besides, the plague built up boundary could be changing with time. The complicated body-fitted geometry method plays a significant role in numerical stability and its eventual accuracy. Hence, for most simulations, either the grids were generated a priori [12], or the computer generated meshes were checked manually to ensure sufficient accuracy of the computational results [13]. In the case of a generated mesh, an a priori knowledge of the artery configuration is required. Therefore, for model blood flow problem, it would be desirable to develop a numerical technique that does not necessarily keeps the mesh conforms to the complicated boundary. One such technique is the immersed boundary (IB) method first introduced by Peskin [15] in his study of blood flow in the human heart. In the IB method, the governing equations are discretized on a fixed Cartesian grid. The wall conditions of a boundary are accounted for by introducing an external force field in the equations of motion, which is designed to ensure that the fluid satisfies the no-slip condition on blood vessel walls. The boundary was modelled as a set of elements linked by springs. Subsequent advanced boundary treatments dealing with complex geometry, moving boundary and even fluid-structure interaction (FSI) can be derived from the IB method [16].

Recently, Fu et al. [17,18] have developed a finite difference lattice Boltzmann method (hereafter designated as FDLBM) that provides a convenient algorithm for setting the boundary condition using a splitting method to solve the discrete lattice Boltzmann (LB) equation. Their FDLBM is capable of simulating flows of Newtonian and non-Newtonian fluids, with (or without) external body forces [17]. Also, the inherent compressibility effect of the conventional LBM [19], which might produce significant errors in some incom-