

Simulations of Boiling Systems Using a Lattice Boltzmann Method

L. Biferale^{1,2,*}, P. Perlekar², M. Sbragaglia¹ and F. Toschi²

¹ *Department of Physics and INFN, University of Tor Vergata, Via della Ricerca Scientifica 1, 00133 Rome, Italy.*

² *Department of Physics and Department of Mathematics and Computer Science, Eindhoven University of Technology, 5600 MB Eindhoven, The Netherlands.*

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Abstract. We report about a numerical algorithm based on the lattice Boltzmann method and its applications for simulations of turbulent convection in multi-phase flows. We discuss the issue of 'latent heat' definition using a thermodynamically consistent pseudo-potential on the lattice. We present results of numerical simulations in 3D with and without boiling, showing the distribution of pressure, density and temperature fluctuations inside a convective cell.

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1 Introduction

Thermal convection, the state of a fluid heated from below and cooled from above, is a ubiquitous phenomenon in nature, present in many industrial, geophysical and astrophysical systems [1]. It is also challenging from the theoretical point of view, due to its extremely rich and different regimes, ranging from intricate pattern formations at moderate temperature jumps from bottom and top plates (i.e. moderate Rayleigh number), to extremely turbulent behaviour where heat transfer and its adimensional definition (i.e. Nusselt number) is dominated by bulk or boundary layer physics (or by both, see e.g. recent reviews [2]). Thermal convection is often studied in its simplest version, the so-called Oberbeck-Boussinesq (OB) approximation, where a single phase – unstratified – fluid is present with constant material properties. Compressibility is also neglected except for buoyancy forces. In many situations some, or all, of the above assumptions breaks down

*Corresponding author. *Email address:* biferale@roma2.infn.it (L. Biferale)

and one speaks about Non-Oberbeck-Boussinesq (NOB) convection. Deviations from OB can arise in many different ways. Two notable cases are (i) the presence of stratification (and/or rotation) arising in many geophysical applications and/or (ii) the presence of boiling, i.e. when the parameter excursions inside the convective cell allows for phase transition inside the flow [3–5].

The equations governing the system are the usual – compressible – Navier-Stokes equations supplied with an equation for the internal energy and for an Equation of State (EoS) defining the non-ideal properties at equilibrium:

$$\begin{cases} \partial_t \rho + \partial_j (\rho u_j) = 0, \\ \partial_t \rho u_i + \partial_j (\rho u_i u_j) = -\partial_i P + \partial_j (\mu (\partial_i u_j + \partial_j u_i)) + g \rho \hat{z}, \end{cases} \quad (1.1)$$

where $\mu = \rho \nu$ is the molecular viscosity, g is the gravity, ρ is the local fluid density and

$$P(\rho, T) = P_0(\rho, T) + P_{NI}(\rho)$$

is the non-ideal pressure. Pressure is fixed by the equation of state and it is made of two terms, the ideal part $P_0(\rho, T) = \rho T$ and the non ideal part which in our LBM system reads: $P_{NI}(\rho) = G \exp(-2/\rho)$ (see below). The equation for the internal energy, $U = c_v T + \int d\rho P_{NI} / \rho^2$ is given by:

$$\rho D_t U + P \partial_j u_j = \kappa \partial_{jj} T, \quad (1.2)$$

where κ is the thermal conductivity. The above equation can also be rewritten in terms of the system temperature in two equivalent ways:

$$\begin{cases} c_p \rho D_t T - \alpha T D_t P = \kappa \partial_{jj} T, \\ c_v \rho D_t T + P_0 \partial_j u_j = \kappa \partial_{jj} T, \end{cases} \quad (1.3)$$

where D_t stands for the material derivative, c_v is the specific heat at constant volume, c_p and $\alpha = -(\partial_T \rho) / \rho$ are the specific heat and compressibility at constant pressure, respectively. The above set of equations tends to the usual OB system when the fluid is considered single phase and incompressible, $\rho = \text{const.}$, $\partial_j u_j = 0$ and both μ, κ are constant [6]. In this proceedings, we report about some technical issues on how to implement the above set of equations using a Lattice Boltzmann Method and on some preliminary applications to study 3D convection under boiling, i.e. allowing for bubbles nucleation and evaporations inside the convective cell.

2 The LB algorithm

In the non ideal gas lattice Boltzmann model, the force experienced by the particles at \mathbf{x} from the particles at \mathbf{x}' is assumed to be in the following form [7, 8, 19]:

$$F(\mathbf{x}, \mathbf{x}') = \mathcal{G}(|\mathbf{x} - \mathbf{x}'|) \psi(\mathbf{x}) \psi(\mathbf{x}'), \quad (2.1)$$