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A Jacobian-Free Newton Krylov Implicit-Explicit Time Integration Method for Incompressible Flow Problems

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Abstract. We have introduced a fully second order **IM**plicit/**EX**plicit (IMEX) time integration technique for solving the compressible Euler equations plus nonlinear heat conduction problems (also known as the radiation hydrodynamics problems) in *Kadioglu et al.*, J. Comp. Physics [22,24]. In this paper, we study the implications when this method is applied to the incompressible Navier-Stokes (N-S) equations. The IMEX method is applied to the incompressible flow equations in the following manner. The hyperbolic terms of the flow equations are solved explicitly exploiting the well understood explicit schemes. On the other hand, an implicit strategy is employed for the non-hyperbolic terms. The explicit part is embedded in the implicit step in such a way that it is solved as part of the non-linear function evaluation within the framework of the Jacobian-Free Newton Krylov (JFNK) method [8,29,31]. This is done to obtain a self-consistent implementation of the IMEX method that eliminates the potential order reduction in time accuracy due to the specific operator separation. We employ a simple yet quite effective fractional step projection methodology (similar to those in [11,19,21,30]) as our preconditioner inside the JFNK solver. We present results from several test calculations. For each test, we show second order time convergence. Finally, we present a study for the algorithm performance of the JFNK solver with the new projection method based preconditioner.

AMS subject classifications: 01-08, 35Q35, 76Dxx

Key words: Incompressible flow, Navier-Stokes equations, IMEX method, JFNK method, preconditioner.

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1 Introduction

In this paper, we present a second order **IM**plicit/**EX**plicit IMEX method for solving the incompressible Navier-Stokes equations. This paper is a continuation of our previous works [22–24] that concern the radiation hydrodynamics problems. [22] considers the low energy density radiation hydrodynamics as in a diffusion approximation limit which can be modelled by the compressible Euler equations plus a nonlinear heat conduction term in the energy equation. [24] considers the more complicated high energy density radiation hydrodynamics as in a diffusion approximation limit which can be modelled by a combination of a hydrodynamical model that resembles to the compressible Euler equations and a radiation energy model that contains separate radiation energy equation with nonlinear diffusion plus coupling terms to the material. In both papers, the hydrodynamics equations (hyperbolic terms) are treated explicitly and an implicit strategy is employed for the diffusion plus source terms. The implicit/explicit (IMEX) technique in [22, 24] is implemented in such a way that the explicit part is solved as part of the nonlinear function evaluation within the framework of the Jacobian-Free Newton Krylov (JFNK) method [8, 29, 31]. In this kind of implementation, there is a continuous interaction between the implicit and explicit blocks. In other words, the improved solutions (in terms of accuracy) at each non-linear iteration (implicit block) are immediately felt by the explicit block, then the improved hydrodynamics solutions (explicit block) are readily available to form the next set of non-linear residuals in the implicit block. We refer the above described IMEX implementation as to "a self-consistent IMEX method" in which all the nonlinearities of the coupled system are converged. These two examples ([22, 24]) represent typical multiple time scale flow problems for which having a second order time convergent algorithm is an important advancement. We remark that we also applied this IMEX methodology to a multi-physics problem that tightly couples the neutron diffusion to a linear mechanics model to simulate experimentally observed certain nuclear fuel material behaviors [25]. In [25], the IMEX method is utilized in such a way that the explicit linear mechanics operator is continuously called within the implicit neutron diffusion solver.

We consider this paper as the prototype for the development of a second order IMEX method for multi-phase flow problems that are modelled by two phase incompressible Navier-Stokes equations. We note that in a typical multi-phase flow, there is a strong coupling between the interface and fluid dynamics, thus it is important to introduce an accurate integration technique that converges all the nonlinearities coming from the coupling of these two different dynamics [27]. This paper will provide us important insights about how to separate operators of the multi-phase flow model self-consistently. The operators of the single phase incompressible Navier-Stokes equations are separated in such a way that the momentum advection terms (hyperbolic terms except the pressure) are solved explicitly, and the viscosity plus pressure terms are treated implicitly. The algorithm is implemented in a similarly way as in [22, 24] that the explicit part is solved as part of the nonlinear function evaluation within the implicit loop.