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Lattice Boltzmann Finite Volume Formulation with Improved Stability

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Abstract. The most severe limitation of the standard Lattice Boltzmann Method is the use of uniform Cartesian grids especially when there is a need for high resolutions near the body or the walls. Among the recent advances in lattice Boltzmann research to handle complex geometries, a particularly remarkable option is represented by changing the solution procedure from the original "stream and collide" to a finite volume technique. However, most of the presented schemes have stability problems. This paper presents a stable and accurate finite-volume lattice Boltzmann formulation based on a cell-centred scheme. To enhance stability, upwind second order pressure biasing factors are used as flux correctors on a D2Q9 lattice. The resulting model has been tested against a uniform flow past a cylinder and typical free shear flow problems at low and moderate Reynolds numbers: boundary layer, mixing layer and plane jet flows. The numerical results show a very good accuracy and agreement with the exact solution of the Navier-Stokes equation and previous numerical results and/or experimental data. Results in self-similar coordinates are also investigated and show that the time-averaged statistics for velocity and vorticity express self-similarity at low Reynolds numbers. Furthermore, the scheme is applied to simulate the flow around circular cylinder and the Reynolds number range is chosen in such a way that the flow is time dependent. The agreement of the numerical results with previous results is satisfactory.

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1 Introduction

Computational fluid dynamics, in its conventional meaning, computes pertinent flow fields in terms of velocity, density, pressure and temperature by numerically solving the Navier-Stokes equations in time and space. At the turn of the 1980s, the Lattice Boltzmann Method (LBM) has been proposed as an alternative approach to solve fluid dynamics problems. The main philosophy of the LBM is to compute the physical reality of a flow field through a microscopic kinetic approach that preserves the hydrodynamic conservation laws [1, 2]. As a different approach from the conventional computational fluid dynamics, the LBM, initially developed from its predecessor the Lattice Gas Automata (LGA) [3], has rapidly evolved into a self-standing research subject and has proven to be an efficient tool for simulating a variety of nontrivial transport phenomena and fluid dynamics problems such as hydrodynamics in porous media, multi-phase or multi-component flows, reactive chemical flow, magneto-hydrodynamics, etc. [4–8]. Because of the broad scope and great potential of its applications, the LBE method has been viewed not only as a novel technique, but also as a new and general approach in the spirit of kinetic theory for the study of complex systems.

The kinetic nature of the LBE leads to several advantages. Pressure field and stress tensor are locally available, with no need of solving any (expensive) Poisson problem. Moreover, non-linearities are local (quadratic dependence of the local equilibrium on the flow field) and the non-localities are linear because advection proceeds along constant, straight lines defined by the discrete particle speeds. This is a very useful property, not shared by the Navier-Stokes equations, in which non-linearity and non-locality come together into the same term, that is, the fluid moves its own momentum along a space-time changing direction defined by the flow speed itself. Finally, the LBE method, the constitutive relations emerge as a result of proper modeling of inter-particle potentials. Several references are available to obtain an entry to the theory and methodology of LBE [9, 10].

One of the crucial ingredients which concurs to the LB efficiency is that particles live on a discrete lattice, thus greatly simplifying the dynamics and the bookkeeping of the method. However, this leads to geometrical stiffness, resulting into a uniform spatial grid. This represents a very severe limitation for many practical applications, particularly for multiscale-type calculations, where selective distribution of the computational degrees of freedom in the "hot" regions is necessary.

Such difficulty may be overcome by decoupling the numerical mesh from the lattice structure, and taking recourse to, one of finite difference or finite element approaches. He et al. [11] proposed an algorithm that adds an interpolation step to the standard LBM and Succi et al. [12] were the first to propose a finite-volume formulation of the lattice Boltzmann equation. Both of these methods require a topologically structured mesh, though the grid points do not have to form a square lattice. Peng et al. [13, 14] proposed a cell-vertex finite volume scheme, which was further developed by Ubertini et al. [15]. This method allows for an arbitrary decomposition of the computational domain into triangular or quadrilateral elements, with otherwise no structural limitations for the mesh.