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Power Laws and Skew Distributions

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Abstract. Power-law distributions and other skew distributions, observed in various models and real systems, are considered. A model, describing evolving systems with increasing number of elements, is considered to study the distribution over element sizes. Stationary power-law distributions are found. Certain non-stationary skew distributions are obtained and analyzed, based on exact solutions and numerical simulations.

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1 Introduction

Power laws are observed in many systems. Particularly, one has to note the critical phenomena in interacting many-particle systems, which are associated with cooperative fluctuations of a large number of microscopic degrees of freedom. The singularities of various quantities in vicinity of the phase transition point are described by the critical exponents. It has been rigorously shown for a class of exactly solved models [1–3], which are mainly the two-dimensional lattice models. For three-dimensional systems, exact results are difficult to obtain and approximate methods are usually used. A review of numerical results, as well as of the applied here standard perturbative renormalization group (RG) methods can be found, e.g., in [4]. An alternative approach has been proposed in [5]. There are also many textbooks devoted to this topic, e.g., [6–9]. A general review of critical phenomena in various systems can be found, e.g., in [10]. Recently, the role of quantum fluctuations in critical phenomena has been reviewed and discussed in [11].

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Goldstone mode power-law singularities are observed also below the critical temperature in some systems, where the order parameter is an n-component vector with $n>1$ (see, e.g., [12–17]). These systems are spin models having $\mathcal{O}(n)$ rotational symmetry in zero external field. This is an interesting example of power law behavior, exhibited by the transverse and longitudinal correlation functions in the ordered phase. Moreover, according to the recent Monte Carlo (MC) simulation results [18–20], it is very plausible that this behavior is described by nontrivial exponents, as predicted in [17].

For a general review, one has to mention that phase transitions described by power laws and critical exponents are observed in variety of systems, such as social, economical, biological systems, as well as vehicular traffic flow, which are often referred in literature as non-physical systems. In particular, traffic flow is a driven one-dimensional system in which, unlike to one-dimensional equilibrium systems, phase transitions are observed. Formation of a car cluster on the road is analogous to aggregation phenomena in many physical systems [21]. The widely used approach in description of the vehicular traffic, as well as the traffic in biological systems such as ants, is the simulation by cellular automata models. One can mention here the famous Nagel-Schreckenberg model [22], which has numerous extensions, e.g., [23–29]. A good review about this topic can be found in [30]. Stochastic fluctuations play an important role here. A new approach to this problem, emphasizing the role of the stochasticity, has been introduced in [31]. The master equation is used here to describe the jam formation on a road as a stochastic one-step process, in which the size of a car cluster is a stochastic variable. The results of this approach have been summarized in the review paper [32], as well as in the recent textbook [33]. The critical behavior, found in a simple traffic flow model considered in [32], is described by the mean-field exponent $\beta=1/2$ for the order parameter (see p. 75 in [32]).

The power laws in critical phenomena have been discussed in [34] in a general context of many other examples, where the power-law distributions emerge. A distinguishing feature of the critical phenomena is the existence of certain length scale, which diverges at specially chosen parameters, i.e., at the critical point. It results in a scale-free or power-law distribution. In some cases, however, no fine tuning of parameters is necessary to observe the critical phenomena. It refers to systems exhibiting the self-organized criticality. Any such system adjusts itself to the critical point due to some dynamical process. The percolation on square lattice have been discussed in [34] as an example of critical phenomena and the forest fire model-as an example of the self-organized criticality. Spin systems with global rotational symmetry could be added here as a different example of the power-law behavior at a divergent length scale. Namely, the correlation length in such systems is divergent at vanishing external field not only at the critical temperature, but also below it. It results in the already mentioned here power-law Goldstone mode singularities.

Apart from the appearance of the divergent length scale, there are also other mechanisms how the power laws emerge. Many examples have been reviewed and discussed in [10, 34–37] pointing out the ubiquitous observation of power law distributions in nature. A tool for analyzing power law distributed empirical data in presented in [36]. A