Commun. Comput. Phys.
doi: 10.4208/cicp.150311.090112a

Vol. **12**, No. 5, pp. 1307-1328
November 2012

A Conservative Lagrangian Scheme for Solving Compressible Fluid Flows with Multiple Internal Energy Equations

Juan Cheng[1,*], Chi-Wang Shu[2] and Qinghong Zeng[3]

[1] *National Key Laboratory of Science and Technology on Computational Physics, Institute of Applied Physics and Computational Mathematics, Beijing 100088, China.*
[2] *Division of Applied Mathematics, Brown University, Providence, RI 02912, USA.*
[3] *Institute of Applied Physics and Computational Mathematics, Beijing 100088, China.*

Received 15 March 2011; Accepted (in revised version) 9 January 2012

Available online 8 May 2012

Abstract. Lagrangian methods are widely used in many fields for multi-material compressible flow simulations such as in astrophysics and inertial confinement fusion (ICF), due to their distinguished advantage in capturing material interfaces automatically. In some of these applications, multiple internal energy equations such as those for electron, ion and radiation are involved. In the past decades, several staggered-grid based Lagrangian schemes have been developed which are designed to solve the internal energy equation directly. These schemes can be easily extended to solve problems with multiple internal energy equations. However such schemes are typically not conservative for the total energy. Recently, significant progress has been made in developing cell-centered Lagrangian schemes which have several good properties such as conservation for all the conserved variables and easiness for remapping. However, these schemes are commonly designed to solve the Euler equations in the form of the total energy, therefore they cannot be directly applied to the solution of either the single internal energy equation or the multiple internal energy equations without significant modifications. Such modifications, if not designed carefully, may lead to the loss of some of the nice properties of the original schemes such as conservation of the total energy. In this paper, we establish an equivalency relationship between the cell-centered discretizations of the Euler equations in the forms of the total energy and of the internal energy. By a carefully designed modification in the implementation, the cell-centered Lagrangian scheme can be used to solve the compressible fluid flow with one or multiple internal energy equations and meanwhile it does not lose its total energy conservation property. An advantage of this approach is that it can be easily applied to many existing large application codes which are based on the framework of solving multiple internal energy equations. Several two dimensional numerical examples for both Euler equations and three-temperature hydrodynamic equations in

*Corresponding author. *Email addresses:* cheng_juan@iapcm.ac.cn (J. Cheng), shu@dam.brown.edu (C.-W. Shu), zeng_qinghong@iapcm.ac.cn (Q. Zeng)

cylindrical coordinates are presented to demonstrate the performance of the scheme in terms of symmetry preserving, accuracy and non-oscillatory performance.

AMS subject classifications: 65M06, 76M20

Key words: Lagrangian scheme, conservative, cell-centered, internal energy equation, compressible fluid flow, three-temperature model.

1 Introduction

The Lagrangian methods, in which the mesh moves with the local fluid velocity, are widely used in many fields for multi-material flow simulations such as in astrophysics and inertial confinement fusion (ICF), due to their distinguished advantage in capturing material interfaces automatically. Such methods can be classified into two types, one is the staggered-grid based Lagrangian method, the other is the cell-centered Lagrangian method. For the staggered-grid based Lagrangian method, the algorithms are built on a staggered discretization in which velocity (momentum) is stored at vertices, while density and internal energy are stored at cell centers. The density/internal energy and velocity are solved on two different control volumes directly, see, e.g., [1, 3, 4, 6, 25, 29]. An artificial viscosity term [5, 7, 29] is usually added to the scheme to prevent spurious oscillations near the discontinuities. As the internal energy equation is discretized directly, this kind of methods usually cannot keep the conservation of the total energy, unless a specially designed compatible construction is used, see [6]. On the other hand, for the cell-centered Lagrangian schemes, density, momentum and total energy are all stored at the cell center and evolved on the same control volume directly. This kind of methods has several advantages such as easiness for remapping when necessary, no need for explicit artificial viscosity for shock capturing and conservation for all the conserved variables including mass, momentum and total energy. In recent years, the cell-centered Lagrangian method has been developed rapidly and many cell-centered schemes have been presented [8, 10–12, 14, 19–24, 26, 28] and they are demonstrated to have good properties such as conservation, accuracy and non-oscillation.

In some hydrodynamic application problems in, e.g., ICF and astrophysics, multiple internal energy equations are involved. For example, the 2D code CHIC [2] applied to simulate the ICF problem involves electron and ion internal energy equations and it is based on a cell-centered Lagrangian scheme discretizing the electron internal energy equation and total energy equation directly. However, most of the existing large codes used in these fields are originally based on the staggered-grid Lagrangian schemes and are designed to solve the internal energy equations directly, e.g., the LASNEX [17], HYDRA [27] and LARED-H [30] codes. As the cell-centered Lagrangian schemes are commonly discretized from the Euler equations in the form of the total energy, it is difficult to use them in these codes without significant modifications. If such modifications are not