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A Preconditioned 3-D Multi-Region Fast Multipole Solver for Seismic Wave Propagation in Complex Geometries

S. Chaillat[1,3,*], J. F. Semblat[2] and M. Bonnet[3]

[1] *Georgia Tech, College of Computing, Atlanta, USA.*
[2] *Université Paris Est, IFSTTAR, Paris, France.*
[3] *POems (CNRS-ENSTA-INRIA), Appl. Math. Dept., ENSTA, Paris, France.*

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Abstract. The analysis of seismic wave propagation and amplification in complex geological structures requires efficient numerical methods. In this article, following up on recent studies devoted to the formulation, implementation and evaluation of 3-D single- and multi-region elastodynamic fast multipole boundary element methods (FM-BEMs), a simple preconditioning strategy is proposed. Its efficiency is demonstrated on both the single- and multi-region versions using benchmark examples (scattering of plane waves by canyons and basins). Finally, the preconditioned FM-BEM is applied to the scattering of plane seismic waves in an actual configuration (alpine basin of Grenoble, France), for which the high velocity contrast is seen to significantly affect the overall efficiency of the multi-region FM-BEM.

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1 Introduction

Due to rapid and steady increase of available computational resources, the simulation of wave propagation in 3D configurations is currently a very active research area. The main advantage of the boundary element method (BEM) is that only the domain boundaries (and possibly interfaces) are discretized, leading to a reduction of the number of degrees of freedom (DOFs), and avoiding cumulative effects of grid dispersion [17, 18]. The BEM is well suited to unbounded-domain idealizations commonly used in seismology, as exact

*Corresponding author. *Email addresses:* `stephanie.chaillat@ensta-paristech.fr` (S. Chaillat), `semblat@lcpc.fr` (J. F. Semblat), `bonnet@lms.polytechnique.fr` (M. Bonnet)

satisfaction of radiation conditions is built into the formulation [3, 20]. However, the standard BEM leads to fully-populated matrices, which result in high computational costs in CPU time ($\mathcal{O}(N^2)$ per iteration using an iterative solver such as GMRES) and memory requirements ($\mathcal{O}(N^2)$), where N denotes the number of DOFs of the BEM model. The appearance of accelerated boundary element (BE) methodologies, allowing complexities far lower than those of traditional BEMs, has dramatically improved the capabilities of BEMs for many areas of application, largely owing to the rapid development of the Fast Multipole Method (FMM) over the last two decades [23]. Such approaches have resulted in considerable solution speedup, memory requirement reduction, and model size increase. The FMM is inherently associated with iterative solvers (usually GMRES), and is known to require $\mathcal{O}(N \log N)$ CPU time per iteration for Helmholtz-type equations [9, 10, 34]. To date, only few studies have been devoted to the FMM in elastodynamics (including [15] for the frequency-domain case, [35] for the time-domain case and [4] for a formulation specialized to surface waves), whereas FMMs for the Maxwell equations have been more extensively investigated, see e.g. [9, 16, 21, 34]. The present authors recently proposed an elastodynamic single-domain FM-BEM formulation which incorporates recent advances of FMM implementations for Maxwell equations [6], with BEM models of size up to $N = \mathcal{O}(10^6)$ run on a single-processor PC, then extended it to multi-domain situations, with emphasis on alluvial-basin configurations, by developing a FMM-based BE-BE coupling approach suitable for 3-D piecewise-homogeneous media [7].

The previous studies [6, 7] revealed that iteration count could significantly hinder the overall efficiency of the elastodynamic FM-BEM, especially for multi-region configurations, and even for problem sizes well within the computational platform's limitations in terms of required memory and CPU cost of a single iteration. This article aims at addressing this issue via a simple preconditioning approach based on an inner-outer GMRES algorithm, whose usefulness is then demonstrated on 3-D numerical examples representative of seismic wave propagation. The proposed preconditioned FM-BEM is then applied to a more realistic seismological configuration, namely the propagation of seismic waves in an alpine basin (Grenoble, France), for which the high velocity contrast is seen to significantly affect the overall performance of the multi-region FM-BEM. The paper is organized as follows. Classical concepts pertaining to elastodynamic BEM and FMM are summarized in Section 2. The preconditioning strategy is presented and demonstrated on numerical examples in Section 3, and applied on the Grenoble site model in Section 4.

2 Standard and fast multipole accelerated BEM

2.1 Single-region boundary element method

Let Ω denote a region of space occupied by an isotropic elastic solid characterized by μ (shear modulus), ν (Poisson's ratio) and ρ (mass density). A time-harmonic motion with circular frequency ω is assumed, and the implicit factor $e^{-i\omega t}$ will be, as usual, omitted throughout. Assuming the absence of body forces, the displacement $\boldsymbol{u}$ is given at an