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# Quantifying Tectonic and Geomorphic Interpretations of Thermochronometer Data with Inverse Problem Theory

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**Abstract.** Thermochronometer data offer a powerful tool for quantifying a wide range of geologic processes, such as the deformation and erosion of mountain ranges, topographic evolution, and hydrocarbon maturation. With increasing interest to quantify a wider range of complicated geologic processes, more sophisticated techniques are needed. This paper is concerned with an inverse problem method for interpreting the thermochronometer data quantitatively. Two novel models are proposed to simulate the crustal thermal fields and paleo mountain topography as a function of tectonic and surface processes. One is a heat transport model that describes the change of temperature of rocks; while the other is surface process model which explains the change of mountain topography. New computational algorithms are presented for solving the inverse problem of the coupled system of these two models. The model successfully provides a new tool for reconstructing the kinematic and the topographic history of mountains.

**AMS subject classifications**: 86A22, 35R37, 35Q80, 86A60

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## 1 Introduction

In recent year, there has been growing interest in developing suitable numerical methods for studying geologic processes. A number of studies have been conducted demon-

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strating how numerical modeling can improve the interpretation of geologic data, for example [3, 9, 13, 14, 17, 21]. Apatite (U-Th)/He thermochronometry has emerged as an important tool for quantifying the cooling history of rocks as they pass through the upper 1-3 km of the crust. The low closure temperature ($\sim 60^\circ$C) of this thermochronometer system has attracted interdisciplinary studies in the Earth science, such as for landform evolution, structural geology, geomorphology, geochemistry, petrology, and geodynamics [2, 7, 8, 22]. In general, thermochronometer data may be interpreted by measuring an age (or other related observables such as fission track lengths or noble gas release) from minerals extracted from rocks at or near the earth's surface. A thermonometer cooling age represents the time since a rock cooled below some effective closure temperature. These ages are influenced by either some events or geologic processes (e.g., erosion, faulting, topographic change, cooling of igneous rocks). In the latter case, which is closely related to our work in this paper, efforts are made to interpret the thermochronometer data to quantify the deformation, erosion, and topographic history of active mountain ranges. More specifically, we present in this paper a novel coupling of topographic evolution and 3D thermal models with inverse problem theory to reconstruct geologic processes. For thermal convection, the physical process is governed by

$$\rho c\left(\frac{\partial T}{\partial t}+\boldsymbol{v}\cdot\nabla T\right)=\nabla\cdot(k_1\nabla T)+\rho H. \tag{1.1}$$

Explanation of each of the terms and parameters will be given later. This equation is a classic heat equation defined on the three dimensional region with moving boundary, considering heat advection, diffusion and radiogenic effect. We also impose suitable boundary conditions based on the underlying physics and geologic setting. For surface process, we have another classic heat type equation, considering transportation by a surface velocity field, diffusivity of hillslope materials, and fluvial processes,

$$\frac{\partial S}{\partial t}=\nabla\cdot(k_2\nabla S)+\boldsymbol{u}\cdot\nabla S+u_3+a\sqrt{Q}\boldsymbol{d}\cdot\nabla S. \tag{1.2}$$

In this study, we do not include glacier erosion in the model because the governing equations are highly nonlinear problem and the evolution of mountain topography in many places can be described to a first order by Eq. (1.2). Our future work will focus on addressing glacial erosion. In Eq. (1.2), $\boldsymbol{v}=(v_x,v_y,v_z)$ is the velocity, $\boldsymbol{u}=(v_x,v_y)$ and $u_3=v_z$. For the inverse problem, the velocity $\boldsymbol{v}$ and surface $S(t,x,y)$ are the unknowns, which need to be reconstructed. The solution of the surface model serves as the moving boundary of the heat process. In our algorithm, we restore the velocity field by solving the inverse heat process model, and apply it as known to the surface model to obtain the initial surface by solving another inverse problem. This is carried out in an iterative fashion. To deal with the inverse problem entangled with a moving boundary, we freeze the boundary for a relatively short time period, by assuming that the mountain range does not change significantly in one thousand years.