## Kernel Density Estimation Based Multiphase Fuzzy Region Competition Method for Texture Image Segmentation

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**Abstract.** In this paper, we propose a multiphase fuzzy region competition model for texture image segmentation. In the functional, each region is represented by a fuzzy membership function and a probability density function that is estimated by a non-parametric kernel density estimation. The overall algorithm is very efficient as both the fuzzy membership function and the probability density function can be implemented easily. We apply the proposed method to synthetic and natural texture images, and synthetic aperture radar images. Our experimental results have shown that the proposed method is competitive with the other state-of-the-art segmentation methods.

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**Key words**: Texture, multiphase region competition, kernel density estimation, fuzzy membership function, total variation.

## 1 Introduction

Image segmentation is a fundamental task in image processing and computer vision. It is aimed to partition an image into a finite number of subregions with homogeneous intensity (color, texture) properties which will hopefully correspond to objects or object parts. Approaches based on the calculus of variation and partial differential equations (PDEs) are powerful in image segmentation. One important reason of their success is that these models are flexible in integrating the geometric information such as shape, length and area. The best known and most influential approaches are Mumford-Shah

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model [20], geodesic active contour [5], geodesic active region [23], Chan-Vese model [7], region competition [31].

In this paper, we focus on the segmentation of texture images. Piecewise smooth/ constant models such as Mumford-Shah model [20] and Chan-Vese model [7] fail in this case. Recently, some variational methods have been proposed to tackle the segmentation of complex textures based on feature extraction techniques [9,14,25,27]. In [9,27], a set of Gabor filters with different scales, orientations and frequencies are applied to the image to create the features to represent texture in the image. Chan et al. in [9] extended the Chan-Vese model to these vector features for texture image segmentation. Because there are many features to be used in the model, the corresponding minimization method can be slow. Savig et al. [27] used the Beltrami framework on the texture features to define a new texture indicator function, and then integrated this function in a combined model of the geodesic active contour [5] and the vectorial Chan-Vese model [7] to segment textural regions. Rousson et al. [25] extracted the texture features by applying an anisotropic diffusion process to the structure tensor. In their segmentation framework, a Gaussian approximation is used for all the features channels, and a nonparametric approximation is used for the first gray image channel. The choice of Gaussian approximation restricts the applicability to limited set of images that satisfy the underlying assumption.

Another kind of variational methods for texture image segmentation is based on region competition. Zhu et al. [31] proposed a region competition method unifying snake, region growing and Bayesian statistics. It is a parametric model since they assume that each region follows a Gaussian distribution. Kim et al. in [16] proposed a nonparametric statistical method for image segmentation using mutual information and curve evolution. However, the above mentioned variational approaches have some practical short-comings. The above energy functionals are not convex in the optimization space (usually the characteristic functions of sets, which is nonconvex collection) and they have local minima. Typically, the gradient decent method is used in the implementation of these models, and are therefore prone to getting stuck in these local minima. Hence these methods are sensitive to initialization. Meanwhile, the implementation of the above models are based on curve evolution and level set approach [22]. The drawback in the level set implementation consists of initializing the active contour in a distance function and re-initializing periodically during the evolution, which is time-consuming.

Based on the observation in the Rudin-Osher-Fatemi [26] model for binary image denoising and Chan-Vese segmentation model, the drawback of leading to local minima comes from the non-convexity of characteristic functions. Recently, Chan et al. [10] proposed to use a "segmentation" variable valued in [0,1] to substitute a characteristic function and obtain a new constrained convex functional such that the global minimizer can be achieved in the segmentation process. To make the algorithm more efficient, Bresson et al. [2] proposed to add another new variable to approximate "segmentation" variable such that the Chambolle's fast dual projection method [6] can be employed. The advantage of this algorithm is that it is fast and easy to implement. There are several works following this idea [3,4,14,18,19]. Mory et al. [18,19] derived the fuzzy region competi-