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Symmetric Energy-Conserved Splitting FDTD Scheme for the Maxwell's Equations

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Abstract. In this paper, a new symmetric energy-conserved splitting FDTD scheme (symmetric EC-S-FDTD) for Maxwell's equations is proposed. The new algorithm inherits the same properties of our previous EC-S-FDTDI and EC-S-FDTDII algorithms: energy-conservation, unconditional stability and computational efficiency. It keeps the same computational complexity as the EC-S-FDTDI scheme and is of second-order accuracy in both time and space as the EC-S-FDTDII scheme. The convergence and error estimate of the symmetric EC-S-FDTD scheme are proved rigorously by the energy method and are confirmed by numerical experiments.

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Key words: Maxwell's equation, ADI method, FDTD, energy-conserved, second-order accuracy, symmetric scheme.

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1 Introduction

For solving multidimensional partial differential equations, specially for parabolic problems, the alternating direction implicit methods (ADI) and the fractional step methods (FS) are very attractive and popular (see, e.g., [6, 8, 26, 27, 29]; and more recent works [5,7,18,20], etc). In computations of Maxwell's equations, many works related to the ADI technique have been studied for reducing the complexities and the large computational costs. For example, Holland in [16] discussed the ADI method combining with Yee's

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scheme for the two-dimensional problems. However, the proposed scheme was difficult to obtain the unconditional stability property for three-dimensional Maxwell's equations. Zheng et al. in [33] first proposed an unconditionally stable ADI-FDTD scheme for the three-dimensional Maxwell's equations with an isotropic and lossless medium. The accuracy and dispersion of this scheme was further studied in [14,32]. Meanwhile, Namiki [23] proposed a kind ADI-FDTD scheme for the Maxwell's equations in two-dimensions. The unconditional stability of the scheme was analyzed in [23,31]. Recently, combining the splitting technique with the staggered Yee's grid, Gao et al. in [11,12] proposed the splitting finite-difference time-domain methods for Maxwell's equations: the S-FDTDI and S-FDTDII schemes for the two-dimensional problems and the S-FDTD and IS-FDTD schemes for three-dimensional problems. All the schemes are efficient and easy to be implemented.

On the other aspect, to keep the original physical features of problems is of great importance in constructing numerical schemes for the long time computations. In the propagation of electromagnetic wave in lossless medium without sources, it is well known that the density of the electromagnetic energy of the wave is constant at different times. The previous ADI or splitting schemes are unconditionally stable and effective for high dimensional problems but often break the property of energy conservation of Maxwell's equations. More recently, in [4] we developed two energy-conserved splitting finite-difference time-domain schemes (EC-S-FDTDI and EC-S-FDTDII), which have important properties: i) Energy-conservation; ii) Unconditional stability; iii) Efficient computation at each time step; iv) Dissipation-free.

Based on the staggered Yee's grid, by applying the splitting technique, the proposed energy-conserved splitting finite-difference time-domain scheme (EC-S-FDTDI) in [4] consists of two stages at each time step and therefore is simple in computational complexity. However, it is only first-order accurate in time. The EC-S-FDTDII scheme in [4] is a three stages scheme, which keeps all the above mentioned properties i) - iv) as the EC-S-FDTDI scheme and is of second-order accuracy in time. The EC-S-FDTDII scheme improves the accuracy of the EC-S-FDTDI scheme but it contains three stages at each time step, i.e., three tri-diagonal systems are to be solved at each time step. By analyzing these two schemes, we note that the EC-S-FDTDI is just symmetric in space but not in time, which may explain its first-order convergence in the time direction. Thus, we propose to modify the EC-S-FDTDI scheme by distinguishing the time steps between the even time step and the odd time step so that the derived scheme is symmetric in the time direction. The new scheme is called the symmetric EC-S-FDTD scheme, which has the same computational complexity as the original EC-S-FDTDI scheme. We prove that the symmetric EC-S-FDTD scheme is energy-conserved, unconditionally stable and dissipative-free. Furthermore, it is shown that the scheme is of second-order accuracy in both time and space. These properties are confirmed by numerical experiments as well.

The remaining of the paper is organized as follows. In Section 2, the conservation properties of Maxwell's equations are introduced and the new symmetric EC-S-FDTD scheme is proposed for the two-dimensional case. In Section 3, the energy conserva-