

Composite Laguerre-Legendre Pseudospectral Method for Exterior Problems

Tian-Jun Wang¹ and Ben-Yu Guo^{2,3,4,*}

¹ Department of Mathematics and physics, Henan University of Science and Technology, LuoYang 471003, China.

² Department of Mathematics, Shanghai Normal University, Shanghai, 200234, China.

³ Division of Computational Science of E-Institute of Shanghai Universities, China.

⁴ Scientific Computing Key Laboratory of Shanghai Universities, China.

Received 1 November 2007; Accepted (in revised version) 14 January 2008

Available online 1 August 2008

Abstract. In this paper, we propose a composite Laguerre-Legendre pseudospectral method for exterior problems with a square obstacle. Some results on the composite Laguerre-Legendre interpolation, which is a set of piecewise mixed interpolations coupled with domain decomposition, are established. As examples of applications, the composite pseudospectral schemes are provided for two model problems. The convergence of proposed schemes are proved. Efficient algorithms are implemented. Numerical results demonstrate the spectral accuracy in space of this new approach.

AMS subject classifications: 65M70, 41A30, 35J25, 35K20

Key words: Composite Laguerre-Legendre interpolation, pseudospectral method for exterior problems.

1 Introduction

Many practical problems require solving partial differential equations defined on exterior domains. Considerable progress has been made in spectral and pseudospectral methods for unbounded domains. A direct and commonly used approach is based on Hermite and Laguerre orthogonal approximations, see, e.g., [4–7,9,17,18,20]. Some authors also developed the mixed Laguerre-Legendre spectral and pseudospectral methods for an infinite strip, see [14, 19]. A challenging problem is how to design spectral and pseudospectral schemes for exterior problems. However, so far, there have been only few literatures concerning the spectral method for exterior problems, see [2, 10, 21].

*Corresponding author. *Email address:* byguo@shnu.edu.cn (B.-Y. Guo)

We now consider exterior problems with a square obstacle $\Omega_0 = \{(x, y) \mid -1 \leq x, y \leq 1\}$. Its boundary $\Gamma = \bigcup_{j=1}^4 \Gamma_j$, where

$$\begin{aligned} \Gamma_1 &= \{(x, y) \mid x = 1, |y| \leq 1\}, & \Gamma_2 &= \{(x, y) \mid |x| \leq 1, y = 1\}, \\ \Gamma_3 &= \{(x, y) \mid x = -1, |y| \leq 1\}, & \Gamma_4 &= \{(x, y) \mid |x| \leq 1, y = -1\}. \end{aligned}$$

In this case, we divide the unbounded domain $\Omega = R^2 / \Omega_0$ into eight subdomains $\Omega_j, 1 \leq j \leq 8$. In other words, $\Omega = \bigcup_{j=1}^8 \Omega_j, 1 \leq j \leq 8$, where

$$\begin{aligned} \Omega_1 &= \{(x, y) \mid x > 1, |y| \leq 1\}, & \Omega_2 &= \{(x, y) \mid x > 1, y > 1\}, \\ \Omega_3 &= \{(x, y) \mid |x| \leq 1, y > 1\}, & \Omega_4 &= \{(x, y) \mid x < -1, y > 1\}, \\ \Omega_5 &= \{(x, y) \mid x < -1, |y| \leq 1\}, & \Omega_6 &= \{(x, y) \mid x < -1, y < -1\}, \\ \Omega_7 &= \{(x, y) \mid |x| \leq 1, y < -1\}, & \Omega_8 &= \{(x, y) \mid x > 1, y < -1\}. \end{aligned}$$

The common boundary of adjacent subdomains Ω_j and Ω_{j+1} are denoted by $\Gamma_{j,j+1}$. In particular, $\Gamma_{8,9} = \Gamma_{8,1}$. Namely,

$$\begin{aligned} \Gamma_{12} &= \{(x, y) \in \Omega, x > 1, y = 1\}, & \Gamma_{23} &= \{(x, y) \in \Omega, x = 1, y > 1\}, \\ \Gamma_{34} &= \{(x, y) \in \Omega, x = -1, y > 1\}, & \Gamma_{45} &= \{(x, y) \in \Omega, x < -1, y = 1\}, \\ \Gamma_{56} &= \{(x, y) \in \Omega, x < -1, y = -1\}, & \Gamma_{67} &= \{(x, y) \in \Omega, x = -1, y < -1\}, \\ \Gamma_{78} &= \{(x, y) \in \Omega, x = 1, y < -1\}, & \Gamma_{81} &= \{(x, y) \in \Omega, x > 1, y = -1\}. \end{aligned}$$

Recently, the authors developed the composite Laguerre-Legendre approximation with its applications to exterior problems with the obstacle Ω_0 , see [13]. But, the pseudospectral method is more preferable in actual computation. In particular, it is convenient to match numerical solutions on the common boundaries $\Gamma_{j,j+1}, 1 \leq j \leq 8$, of adjacent subdomains, and is easy to deal with nonlinear terms. Moreover, we can use the pseudospectral method coupled with finite element methods, for various exterior problems with more complex geometry.

In this paper, we investigate the composite Laguerre-Legendre pseudospectral method coupled with domain decomposition for exterior problems with the obstacle Ω_0 . We shall use the mixed Laguerre-Legendre interpolations on the subdomains $\Omega_j, j = 1, 3, 5, 7$, and the two-dimensional Laguerre interpolations on the subdomains $\Omega_j, j = 2, 4, 6, 8$. We also introduce certain specific basis functions, induced by the scaled Laguerre functions and the Legendre polynomials, so that the global numerical solutions belong to the space $H^1(\Omega)$ and possess high accuracy. In order to derive precise estimates of numerical solutions, we develop the composite Laguerre-Legendre interpolations on the whole domain Ω , which play important roles in the related pseudospectral methods for various exterior problems. As examples of applications, we consider two model exterior problems. The convergence of proposed pseudospectral schemes are proved. Efficient implementations are described. The corresponding linear discrete systems are symmetric and sparse, which can be resolved easily. Especially, they are suitable for parallel computation. Numerical results demonstrate the spectral accuracy in space of this