

## Numerical Simulation of Wake-Field Acceleration Using an Eulerian Vlasov Code

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**Abstract.** We study the generation of nonlinear plasma wake fields by intense laser pulses, using an Eulerian code for the numerical solution of the fully relativistic one-dimensional (1D) Vlasov-Maxwell equations. The examination of the phase-space of the distribution function allows to study without numerical noise aspects of the particle acceleration by the wake-field generated by intense laser pulses, in the very low density regions of the phase-space. We study the effect of the thermal spread on the existence of accelerated beams, and we compare between results obtained from a circularly polarized wave and a linearly polarized wave.

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**Key words:** Eulerian code, Vlasov-Maxwell, wake-field, plasma.

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### 1 Introduction

Large amplitude wake fields can be produced by propagating ultrahigh power, short laser pulses in underdense plasmas. When the laser power is high enough, the electron oscillation (quiver) velocity becomes relativistic, and large amplitude wake fields are generated which support acceleration gradients much greater than those obtained in conventional linear accelerators. Some important aspects of this problem and other nonlinear problems related to large amplitude laser-plasma interactions have been discussed using the cold relativistic fluid equations [1, 2] (see also the review article in [3]).

Numerical simulations however remain the only alternative to study the kinetic effects in this highly relativistic and highly nonlinear problem. Kinetic effects (e.g., particle trapping and acceleration) in short-pulse laser-plasma interactions are often simulated numerically using particle-in-cell (PIC) codes. However several numerical effects in PIC codes can lead to phase-space errors and unphysical numerical heating in the simulation,

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and hence the detailed phase-space structure and kinetic effects will be poorly approximated in the simulation. It was indeed reported in [4] that the results obtained by the PIC codes show a momentum spread inside the laser pulse which is excessively and unphysically large. At a high laser intensity, this can lead to spurious trapping of erroneously large levels. The correction of this momentum spread error may require unacceptably high resolution in a PIC code, especially if we want to look beyond the bulk fields for phenomena which depends on the details of the phase-space, especially in the low density regions. A warm fluid model has been presented to study laser-plasma interactions in [4], and in the results reported the bulk fields were insensitive to the details of the distribution functions, for the set of parameters used. This was explained by the fact the Lorentz force was much more important than the pressure force at the temperatures considered. However, many processes of interest, like the trapping and acceleration of a beam, should depend on the details of the phase-space.

We study in the present work the problem of the laser wake-field acceleration by using an Eulerian Vlasov code for the direct numerical solution in phase-space of the 1D relativistic Vlasov-Maxwell equations. Eulerian Vlasov codes have been successfully applied in recent years to study several problems in plasma physics, especially problems associated with wave-particle interaction and stimulated Raman scattering [5]. Interest in Eulerian grid-based Vlasov solvers arise from the very low noise level associated with these codes, which allows accurate representation of the low density regions of the phase-space. This is obviously important if the physics of interest is in the low density region of phase-space or in the high energy tail of the distribution function, as is the case in the present problem. In the laser wake-field accelerator concept, a correctly placed trailing electron bunch can be accelerated by the longitudinal electric field and focused by the transverse electric field of the wake plasma waves. A fully nonlinear 1D relativistic Vlasov-Maxwell model to study the self-consistent interaction of intense laser pulses with plasmas can be found for instance in [6]. A characteristic parameter of a high power laser beam is the normalized vector potential

$$|\vec{\alpha}_\perp| = |e\vec{A}_\perp / M_e c^2| = \alpha_0,$$

where  $\vec{A}_\perp$  is the vector potential,  $e$  and  $M_e$  the electronic charge and mass respectively, and  $c$  the speed of light. We are interested in the regime  $\alpha_0 \geq 1$ .

A problem related to the laser wake-field accelerator concept is the plasma wake-field accelerator, where the plasma responds to the self-fields of a driving electron beam, instead of the ponderomotive forces of the laser pulse. The problem of the plasma wake-field accelerator has been studied using an Eulerian Vlasov code to solve the 1D relativistic Vlasov-Maxwell equations [7]. The numerical technique was based on a fractional step method similar to what has been presented in [8,9]. This numerical approach differs from the one we use in the present work, which consists of integrating the Vlasov equation along its characteristics in two dimensions, using a tensor product of cubic  $B$ -splines for the interpolation along the characteristics, without applying a fractional step technique [10]. A kinetic equation for the ions is included in the simulations.