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A Third Order Conservative Lagrangian Type Scheme on Curvilinear Meshes for the Compressible Euler Equations†

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Abstract. Based on the high order essentially non-oscillatory (ENO) Lagrangian type scheme on quadrilateral meshes presented in our earlier work [3], in this paper we develop a third order conservative Lagrangian type scheme on curvilinear meshes for solving the Euler equations of compressible gas dynamics. The main purpose of this work is to demonstrate our claim in [3] that the accuracy degeneracy phenomenon observed for the high order Lagrangian type scheme is due to the error from the quadrilateral mesh with straight-line edges, which restricts the accuracy of the resulting scheme to at most second order. The accuracy test given in this paper shows that the third order Lagrangian type scheme can actually obtain uniformly third order accuracy even on distorted meshes by using curvilinear meshes. Numerical examples are also presented to verify the performance of the third order scheme on curvilinear meshes in terms of resolution for discontinuities and non-oscillatory properties.

AMS subject classifications: 65M99

Key words: Lagrangian type scheme, high order accuracy, conservative scheme, curvilinear mesh, WENO reconstruction, compressible Euler equations.

1 Introduction

In numerical simulations of multidimensional fluid flow, there are two typical choices: a Lagrangian framework, in which the mesh moves with the local fluid velocity, and an Eulerian framework, in which the fluid flows through a grid fixed in space. More

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generally, the motion of the grid can also be chosen arbitrarily, this method is called the Arbitrary Lagrangian-Eulerian method (ALE; see, e.g., [1,7,14]).

Lagrangian methods are widely used in many fields for multi-material flow simulations such as astrophysics and computational fluid dynamics (CFD), due to their distinguished advantage of being able to capture the material interface sharply. In the past years, many efforts have been made to develop Lagrangian methods. Some algorithms are developed from the nonconservative form of the Euler equations, see, e.g., [10,13,22], while others start from the conservative form of the Euler equations, e.g. those in [1,4,11,12].

In our previous paper [3], we developed a class of Lagrangian type schemes on quadrilateral meshes for solving the Euler equations which are based on the high order essentially non-oscillatory (ENO) reconstruction. Advantages for the schemes in [3] include their conservativity for the density, momentum and total energy, their essentially non-oscillatory performance, parameter-free implementation, and their formal high order accuracy both in space and time. However, we also found that our third order Lagrangian type scheme could only achieve second order accuracy on two dimensional distorted Lagrangian meshes. We attributed this phenomenon to a fundamental problem in the formulation of the Lagrangian schemes. Since in a Lagrangian simulation, each cell represents a material particle, its shape may change with the movement of fluid. Therefore, a cell with an initially quadrilateral shape may not keep its shape as a quadrilateral at a later time. It usually becomes a curved quadrilateral. Thus if during our Lagrangian simulation the mesh is always kept as quadrilateral with straight-line edges, this approximation of the mesh will bring second order error into the scheme. Finally we made a conclusion that for a Lagrangian type scheme in multi-dimensions, it can be at most second order accurate if curved meshes are not used. Meanwhile, we also predicted that our scheme can be extended to higher than second order accuracy if curvilinear meshes are used. In this paper, we explore curvilinear meshes to demonstrate our claim stated above. We will develop a third order scheme on curved quadrilateral meshes in two space dimensions. The reconstruction is based on the high order WENO procedure [8,9] but with simpler linear weights. The accuracy test and some non-oscillatory tests are presented to verify our claim. The scheme can also be extended to higher than third order accuracy if a higher order approximation is used on both the meshes and the discretization of the governing equations.

An outline of the rest of this paper is as follows. In Section 2, we describe the individual steps of the third order Lagrangian type scheme on curvilinear meshes in two space dimensions. In Section 3, numerical examples are given to verify the performance of the new Lagrangian type method. In Section 4 we will give concluding remarks.

2 The third order conservative Lagrangian type scheme on curvilinear meshes