

A General Moving Mesh Framework in 3D and its Application for Simulating the Mixture of Multi-Phase Flows

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Abstract. In this paper, we present an adaptive moving mesh algorithm for meshes of unstructured polyhedra in three space dimensions. The algorithm automatically adjusts the size of the elements with time and position in the physical domain to resolve the relevant scales in multiscale physical systems while minimizing computational costs. The algorithm is a generalization of the moving mesh methods based on harmonic mappings developed by Li et al. [J. Comput. Phys., 170 (2001), pp. 562-588, and 177 (2002), pp. 365-393]. To make 3D moving mesh simulations possible, the key is to develop an efficient mesh redistribution procedure so that this part will cost as little as possible comparing with the solution evolution part. Since the mesh redistribution procedure normally requires to solve large size matrix equations, we will describe a procedure to decouple the matrix equation to a much simpler block-tridiagonal type which can be efficiently solved by a particularly designed multi-grid method. To demonstrate the performance of the proposed 3D moving mesh strategy, the algorithm is implemented in finite element simulations of fluid-fluid interface interactions in multiphase flows. To demonstrate the main ideas, we consider the formation of drops by using an energetic variational phase field model which describes the motion of mixtures of two incompressible fluids. Numerical results on two- and three-dimensional simulations will be presented.

AMS subject classifications: 65M20, 65M50, 65M60

Key words: Moving mesh methods, multi-phase flows, unstructured tetrahedra, phase field model, Navier-Stokes equations, finite element method.

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1 Introduction

Several moving mesh techniques have been introduced in the past, in which the most advocated method is the one based on solving elliptic PDEs first proposed by Winslow [31]. Winslow's formulation requires the solution of a nonlinear, Poisson-like equation to generate a mapping from a regular domain in a parameter space Ω_c to an irregularly shaped domain in physical space Ω . Brackbill and Saltzman [5] formulated the grid equations in a variational form to produce satisfactory mesh concentration while maintaining relatively good smoothness and orthogonality. Their approach has become one of the most popular methods used for mesh generation and adaptation. Dvinsky [15] suggests the possibility that harmonic function theory may provide a general framework for developing useful mesh generators. His method can be viewed as a generalization and extension of Winslow's method. However, unlike most other generalizations which add terms or functionals to the basic Winslow grid generator, his approach uses a single functional to accomplish the adaptive mapping. The critical points of this functional are *harmonic maps*. Meshes obtained by Dvinsky's method enjoy desirable properties of harmonic maps, particularly regularity and smoothness.

Motivated by the work of Dvinsky, a moving mesh finite element strategy based on harmonic mapping was proposed and studied by Li et al. in [19]. The key idea is to construct the harmonic map between the physical space and a parameter space by an iterative procedure. In [20], a moving mesh method based on the minimization of the mesh energy is proposed which seems having big potential for simulations in high space dimensions. More precisely, in the mesh points re-distribution step, we solve an optimization problem with some appropriate constraints, which is in contrast to the traditional method of solving the Euler-Lagrange equation directly. The key idea of this approach is to treat the interior and boundary grids as a whole, rather than considering them separately.

In this paper, we present an adaptive moving mesh algorithm for meshes of unstructured tetrahedra in three dimensions, which is a generalization of the moving mesh methods based on harmonic mappings developed by Li et al. in [19, 20]. To make 3D moving mesh simulations possible, the key is to provide a more efficient mesh redistribution procedure so that this part will cost very little comparing with the solution evolution part. Since the mesh re-distribution procedure normally requires to solve large scaled algebraic systems (arising from discretizing the Euler-Lagrange equations or the minimization problem), we will describe a procedure to decouple the matrix equation to a much simpler block-tridiagonal type which can be solved by multi-grid solvers very efficiently. The proposed algebraic multigrid solver deals with the nonlinear constraints locally in the smoothing operation, so that the multigrid procedure is comparative to some point-wise Gauss-Seidel iterations. This allows us to avoid solving a saddle point problem.

To demonstrate the performance of the proposed 3D moving mesh strategy, the algorithm is implemented in finite element simulations of deformable droplet and fluid-fluid interface interactions for multi-phase flows. The flow we consider has discontinuous