COMMUNICATIONS IN COMPUTATIONAL PHYSICS
Vol. **3**, No. 3, pp. 603-640

Commun. Comput. Phys.
March 2008

Modelling of Propagating Shear Waves in Biotissue Employing an Internal Variable Approach to Dissipation

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Received 7 February 2007; Accepted (in revised version) 28 July 2007

Communicated by Wei Cai

Available online 30 October 2007

Abstract. The ability to reliably detect coronary artery disease based on the acoustic noises produced by a stenosis can provide a simple, non-invasive technique for diagnosis. Current research exploits the shear wave fields in body tissue to detect and analyze coronary stenoses. The methods and ideas outlined in earlier efforts [6] including a mathematical model utilizing an internal strain variable approximation to the quasi-linear viscoelastic constitutive equation proposed by Fung in [19] is extended here. As an initial investigation, a homogeneous two-dimensional viscoelastic geometry is considered. Being uniform in θ, this geometry behaves as a one dimensional model, and the results generated from it are compared to the one dimensional results from [6]. To allow for different assumptions on the elastic response, several variations of the model are considered. A statistical significance test is employed to determine if the more complex models are significant improvements. After calibrating the model with a comparison to previous findings, more complicated geometries are considered. Simulations involving a heterogeneous geometry with a uniform ring running through the original medium, a θ-dependent model which considers a rigid partial occlusion formed along the inner radius of the geometry, and a model which combines the ring and occlusion are presented.

AMS subject classifications: 74D10, 35L45, 62P10

Key words: Viscoelasticity, partial differential equations, shear waves, biotissue.

1 Introduction

Coronary artery disease (CAD), also known as coronary heart disease (CHD), is the most common form of heart disease. The National Institute of Health estimates that approximately seven million Americans suffer from the disease, and roughly 500,000 deaths per

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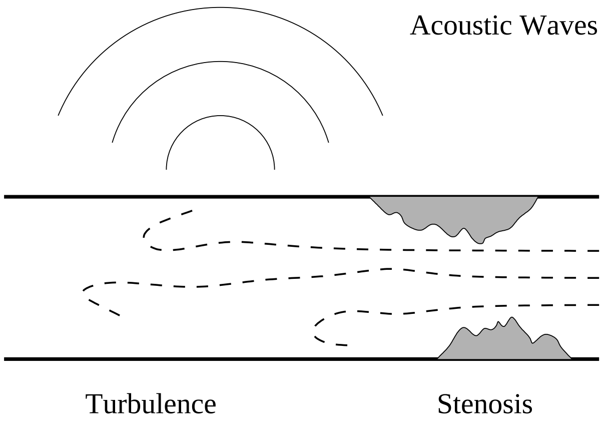

Figure 1: Turbulent blood flow generated by a stenosis.

year can be attributed to CHD [3]. It is caused by atherosclerosis, the gradual buildup of plaque (cholesterol, calcium, and platelets) within the artery. This accumulation of plaque, known as a *stenosis*, restricts the flow of blood, leading to a decrease in the oxygen supply to the heart muscle. The end result of an arterial stenosis is permanent damage to the heart muscle, possibly leading to death. Because CAD affects so many people in the United States, and worldwide, its detection and treatment is a matter of significant interest.

Current detection techniques include the angiogram which is a reliable, yet expensive, invasive technique and also prone to interobserver variability (see e.g., [20,33]). Ultra-fast CT techniques are also employed; this is a non-invasive imaging technique effective in detecting and scoring the severity of calcium deposits in the coronary arteries. Unfortunately, CT testing equipment is very expensive, and it only detects calcium deposits and not the soft plaques that make up many of the most dangerous lesions.

A plausible alternative to the angiogram and CT scan for detection of stenoses utilizes the detection of acoustic waves propagating from the stenosis. It is well known that arterial stenoses produce sounds due to turbulent blood flow in partially occluded arteries. In principle, turbulent normal wall forces exist at and downstream from an arterial stenosis. These wall forces, which are extremely small, exert a pressure on the wall of the artery which then causes a small displacement in the surrounding body tissue (see Fig. 1). The vibrations of the surrounding body tissues, which occur in two forms, a compressional wave and a shear wave, produce sounds [39]. In larger arteries such as the carotid arteries, these acoustic sounds can be detected by physicians using a stethoscope. However, detecting acoustic signals in smaller arteries deep inside the body has proved difficult for two reasons: these acoustic noises attenuate significantly as they travel through the intervening tissues, and many complex sounds within the body can overwhelm conventional acoustic detection systems.

During the late 1990's, MedAcoustics Inc., a company financed by venture capital,