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REVIEW ARTICLE

Finite Size Effects in the Dynamics of Opinion Formation

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Abstract. For some models of relevance in the social sciences we review some examples in which system size plays an important role in the final outcome of the dynamics. We discuss the conditions under which changes of behavior can appear only when the number of agents in the model takes a finite value. Those changes of behavior can be related to the apparent phase transitions that appear in some physical models. We show examples in the Galam's model of opinion transmission and the Axelrod's model of culture formation stressing the role that the network of interactions has on the main results of both models. Finally, we present the phenomenon of system-size stochastic resonance by which a forcing signal (identified as an advertising agent) is optimally amplified by a population of the right (intermediate) size. Our work stresses the role that the system size has in the dynamics of social systems and the inappropriateness of taking the thermodynamic limit for these systems.

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Contents

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1 Introduction

In statistical physics we are used to taking routinely the thermodynamic limit in which the number of constituents (or, more precisely, the number of degrees of freedom) N tends to infinity [1]. This is necessary in many aspects, such as determining the validity of the statistical approach, the equivalence between ensembles, the existence of phase transitions with symmetry breaking and discontinuities in the order parameters or their derivatives, etc. When we apply the methods of statistical physics to explain the properties of macroscopic matter, it is clear that the number N is always finite, but very large. A good estimative of its order of magnitude is the Avogadro number $N_0 = 6.023\times 10^{23}$. Consequently, the nowadays widely used computer simulations of physical systems always struggle to get to larger and larger systems with the continuous increasing demand in computer resources. In many cases as, for instance, when trying to predict the thermodynamical properties near a critical point, it is essential to use the well developed techniques of finite size scaling [2] in order to extrapolate the results to $N\to\infty$.

When applying the same tools of statistical physics to problems of interest in social sciences [3–5], we have to take into account that the number of individuals or agents considered can never be that large. In most cases, realistic values of N range in the hundreds or thousands, reaching at most a few billion. The thermodynamic limit might no be justified in this case, as the results in that limit can vary with respect to those of finite-size systems. Furthermore, new and interesting phenomena can appear depending on the number of individuals or agents considered. In this paper we will review some finite-size related effects that have been recently found in some models of interest in social systems and that exemplify cases in which the thermodynamic limit does not capture the essentials of the system's behavior.

The outline of the paper is as follows: in the following Section 2.1, we will explain the concept of *apparent phase transitions* and *pseudo-critical points* by reviewing a physical example in which a clearly observable change of behavior disappears in the thermodynamic limit. We will then show in the following sections that apparent phase transitions also appear in two widely used models: the Galam model for opinion spreading (in Section 2.2) and the Axelrod model of culture formation (in Section 2.3); finally, Section 3 is devoted to the phenomenon of system-size stochastic resonance in a model for opinion formation that takes into account external influences. A brief summary and concluding remarks are presented in the final Section 4.

2 Apparent phase transitions

2.1 Pseudo-critical temperatures

A true phase transition can only appear in the thermodynamic limit [1]. This limit is needed for the singularities inherent to the transition (for instance, the divergences in the critical point of a continuous, second order, phase transition or the discontinuities in a