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A POSTERIORI ERROR ESTIMATORS FOR NONCONFORMING APPROXIMATION

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Abstract. In this paper, an alternative approach for constructing an a posteriori error estimator for non-conforming approximation of scalar elliptic equation is introduced. The approach is based on the usage of post-processing conforming finite element approximation of the non-conforming solution . Then, the compatible a posteriori error estimator is defined by the local norms of difference between the nonconforming approximation and conforming post-processing approximation on the element plus an additional residual term. We prove in general dimension the efficiency and the reliability of these estimators, without Helmholtz decomposition of the error, nor regularity assumption on the solution or the domain, nor saturation assumption. Finally explicit constants are given, which prove that these estimators are robust in suitable norms

Key Words. Nonconforming finite elements, a posteriori error estimators.

1. 1. Introduction

During the last 15-20 years a big amount of work has been devoted to a posteriori error estimation problem, i.e computing reliable bounds on the error of given numerical approximation to the solution of partial differential equations using only numerical solution and the given data. In order to be operating the a posteriori error estimator should be neither under nor overestimate the error. Most of the work concern the conforming finite element methods [8] and there is no much papers dealing with the nonconforming approximations (see e.g [5][4]). It turned out that in this case some extra terms have to be added to well-know a posteriori error estimator used for conforming case. In [5][4], these extra terms are the jumps across the element edges of the tangential derivatives of the finite element approximation with respect to element edges. In [2], other approach for constructing an a posteriori error estimator is considered which is based on the solution of two local sub-problems.

In this paper, an alternative approach is presented which is based on the usage of post-processing conforming finite elements approximation $\hat{u}_h$ of the nonconforming solution u_h. Then, the compatible a posteriori error estimator is defined as the local norms of $u_h - \hat{u}_h$ on the element plus an additional residual term. We prove in general dimension, without Helmholtz decomposition of the error, nor regularity of the solution or the domain, nor saturation assumption, the efficiency and the reliability of our estimator. Since most known a posteriori error estimates yield two-sided bounds on the error which contain multiplicative constants, an explicit knowledge of such constants is mandatory for a correct calibration of the a posteriori error estimates. The norms of the quasi-interpolation operator have recently been

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estimated explicitly in [7]. In this paper we give explicitly such constants for our estimators.

In the next section, we give some technical lemmas we need in order to estimate the constants in the upper bound of the error. The estimators are introduced in section 3 and the proof of their efficiency and reliability is given.

In order to avoid technical difficulties and to make the underlying ideas as clear as possible, we consider the simple elliptic model problem:

$$\begin{cases} \text{Find } u \text{ such that} \\ -div(A.\nabla u) = f, \quad \text{on } \Omega \\ u = 0, \quad \text{in } \partial\Omega \end{cases},$$

where Ω is an opened bounded polygonal domain in $I\!R^d$ $(d = 2, 3)$ and A is piecewise constant, elliptic and symmetric matrix.

Let $\mathcal{T}_h$ be a conforming triangulation of Ω by triangles or tetrahedrons but nor regular in the sense of Ciarlet [3], we denoted by E_I the set of interior edges (faces)and by E_f the set of all edges (faces) included in $\Gamma := \partial\Omega$. Let V_h be the lowest order nonconforming finite element space defined in [3] :

$$V_h = \{v_h \in L^2(\Omega); \forall T \in \mathcal{T}_h,\ v_{h|T} \in P_1(T), \forall e \in E_I,\ \textstyle\int_e [v_h]_e d\sigma = 0 \\ \text{and } \ \forall e \in E_f,\ \displaystyle\int_e v_h d\sigma = 0\}$$

where $[.]_e$ denotes the jump of the concerned function across e.

We consider the following discrete problem

$$\begin{cases} \text{Find } u_h \in V_h \ \text{ such that} \\ \forall v_h \in V_h; \quad \displaystyle\sum_{T \in \mathcal{T}_h} \int_T A.\nabla u_h.\nabla v_h dx = \int_\Omega \hat{f} v_h dx, \end{cases}$$

where $\hat{f}$ is an approximation of f.

2. Some technical Lemmas

Let us introduce the norms $\|A^{1/2}.\|$ and $\|A^{-1/2}.\|$ defined by :

$$\forall x \in I\!R^d, \quad \|A^{1/2}x\|^2 =< Ax, x > \quad \text{and} \quad \|A^{-1/2}x\|^2 =< A^{-1}x, x > .$$

For all $T \in \mathcal{T}_h$, we denote by E_T the set of edges (faces) of T and we set :

$$h_{A,T} = \max_{x,y \in T} \|A^{-1/2}(x - y)\|,$$

and

$$\rho_{T,A} = 2 \sup_{x \in T} \inf_{y \in \partial T} \|A^{-1/2}(x - y)\|.$$

In the sequel of this paper, we set

$$\mu = \inf_{0 \le \epsilon < 1/2} \frac{(\int_0^1 (1-t)^{2\epsilon} min(t^{-d}, (1-t)^{-d}) dt)^{1/2}}{(1 - 2\epsilon)^{1/2}}.$$

Let us remark that :

$$\mu \le (\int_0^1 min(t^{-d}, (1-t)^{-d}) dt)^{1/2} = (2\frac{2^{d-1} - 1}{d - 1})^{1/2} = d^{1/2}, \quad d = 2, 3.$$

First, we have the lemma