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The Locally Conservative Galerkin (LCG) Method — a Discontinuous Methodology Applied to a Continuous Framework

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Abstract. This paper presents a comprehensive overview of the element-wise locally conservative Galerkin (LCG) method. The LCG method was developed to find a method that had the advantages of the discontinuous Galerkin methods, without the large computational and memory requirements. The initial application of the method is discussed, to the simple scalar transient convection-diffusion equation, along with its extension to the Navier-Stokes equations utilising the Characteristic Based Split (CBS) scheme. The element-by-element solution approach removes the standard finite element assembly necessity, with an face flux providing continuity between these elemental subdomains. This face flux provides explicit local conservation and can be determined via a simple small post-processing calculation. The LCG method obtains a unique solution from the elemental contributions through the use of simple averaging. It is shown within this paper that the LCG method provides equivalent solutions to the continuous (global) Galerkin method for both steady state and transient solutions. Several numerical examples are provided to demonstrate the abilities of the LCG method.

AMS subject classifications: 35D99, 65-05, 76D99, 76Z05

Key words: Explicit local flux conservation, element-by-element solution, local conservation, LCG, convection-diffusion, CBS, artificial compressibility, incompressible flow.

1 Introduction

The locally conservative Galerkin (LCG) method, introduced in 2004 [1], has been developed and employed in an increasing number of applications. It is however important to note that its potential has not yet been fully realised. In this paper an overview

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of the method is provided along with several numerical examples to demonstrate its ability.

The conservation of the Galerkin finite element method has been investigated by many researchers [2–7] and the method has been shown to be globally conservative if Neumann boundary conditions are imposed. Local conservation is seen as a highly valued property possessed by both finite volume methods and discontinuous Galerkin (DG) finite element methods. Local (element-wise) conservation may be of advantage when considering problems involving discontinuities or interfaces in problems such as porous-medium interfaces and fluid structure interaction.

In order to achieve a timely solution, the use of parallel computation is a desirable property for a method to possess. In order to parallelise the global Galerkin method domain decomposition techniques are required [8–10]. An alternative to decomposing the domain would be to use an element-by-element solution scheme. Such a scheme is already decomposed into multiple subdomains and as such is more applicable to parallel computation.

To rectify some of the inherent drawbacks of the continuous (or global) Galerkin (GG) method, the discontinuous Galerkin method (DGM) [11–22] has been developed. The DGM allows for element by element solution and can therefore be easily parallelised [23, 24]. Discontinuous methods can also utilise varying orders of approximation to locally capture a more refined solution [25]. Use of DG methods within industry is often hampered by the large CPU and memory requirements, due to the requirements of storing multiple solutions at a node as well as solving for additional flux variables. Therefore, researchers have been seeking a DG method with the structure of a continuous Galerkin (CG) method [26].

The approach adopted within this paper allows for the introduction of the interface fluxes within the continuous (global) framework [1, 27, 28], rather than adopt a discontinuous Galerkin method to a continuous Galerkin framework. This method can be readily adapted to existing industrial codes with a minimum of modification. The adoption of the interface fluxes does not necessarily have to break the shape function spaces but can be constructed within the continuous framework. The proposed method, corrected LCG, is identical to both the standard and stabilised versions of the continuous Galerkin method for internal nodes. The LCG method and the CG method are not identical on the global boundaries due to the nature of the CG method itself. The LCG method is implicitly globally conservative and does not require the extra calculations CG would require to be globally conservative. The LCG method utilises an element-by-element solution approach removing the standard finite element assembly, with a edge (2D) or face (3D) flux providing continuity between these elemental subdomains. This removal of the standard assembly allows the elemental matrices to be computed, inverted and stored at the pre-processing stages of a computation. This is possible since often these matrices are functions of an element property, such as volume. In an Eulerian frame of reference this property does not change, therefore control over individual elements gives a great deal of freedom to optimise memory requirements. Investigation has shown that the majority of LCG forms, especially the implicit