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A POSTERIORI ERROR ESTIMATES FOR LOCAL C^0 DISCONTINUOUS GALERKIN METHODS FOR KIRCHHOFF PLATE BENDING PROBLEMS*

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Abstract

We derive some residual-type a posteriori error estimates for the local C^0 discontinuous Galerkin (LCDG) approximations ([31]) of the Kirchhoff bending plate clamped on the boundary. The estimator is both reliable and efficient with respect to the moment-field approximation error in an energy norm. Some numerical experiments are reported to demonstrate theoretical results.

Mathematics subject classification: 65N15, 65N30.
Key words: Kirchhoff bending plate, Discontinuous Galerkin method, A posteriori error analysis.

1. Introduction

Over the past two decades, discontinuous Galerkin (DG) methods have been attracting considerable attention as a flexible and efficient computational scheme for many kinds of problems arising in physics and engineering, including linear and nonlinear hyperbolic problems, Navier-Stokes equations, convection-dominated diffusion problems and Maxwell equations; see e.g., [21] and the references therein. A very extensive and thorough study has been done in solving second-order equations/systems by DG methods ([2, 11, 16, 18, 20], to name but a few).

For fourth order problems, e.g., the biharmonic equation and the Kirchhoff plate bending problem, the research dates back to the 1970s [3, 4] and focuses on the interior penalty (IP) methods [10,23,24,27,34–36,39]. Based on the ideas in [16,20] for second order problems, there have developed in [31] a general framework, covering methods in [10,41], of constructing stable C^0 discontinuous Galerkin (CDG) methods for solving the Kirchhoff plate bending problem.

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With some parameter, precisely C_{22}, taken to be zero in determining numerical traces, a so-called local C^0 discontinuous Galerkin (LCDG) method follows, which may be viewed as an extension of the local discontinuous Galerkin (LDG) method in [16, 20] to fourth order elliptic problems. In addition, optimal-order a priori error estimates for the displacement field in certain discrete energy norm and H^1-norm are established in [31]. It is also worth mentioning that in a recent work [19] a new DG method, called LDG-Hybridizable Galerkin method, is applied to the biharmonic problem and the a priori error estimates are derived. Although this method is formulated as a first-order system approximating four variables simultaneously, the globally coupled degrees of freedom are only two of them on the faces of the elements so that the implementation is very efficient.

As is known to all, DG methods are well-suited for use in adaptive algorithms, which are usually based on a posteriori error estimates measuring actual discretization errors without recourse to the exact solution and providing information on where a local refinement is required. There have been great and rapid advances in the theory of a posteriori error analysis for second order elliptic problems. In [5], Becker, Hansbo and Larson derived a residual-based reliable error estimate in certain mesh-dependent energy norm for IP methods with the help of the Helmholtz decomposition. Later with a similar technique applied, a reliable a posteriori error estimate for the LDG method was presented in [13]. Further results concerning the issue are available in [1, 14, 15, 29, 32, 37].

Very recently, Hansbo and Larson [28] developed a reliable a posteriori bound of the energy-norm displacement error for a C^0 interior penalty method for the Kirchhoff bending plate by means of a Helmholtz decomposition of second order tensor fields due to Beirão da Veiga et al [6]. We remark in passing that the decomposition had been originally proposed to construct the residual-based a posteriori error estimate of the nonconforming Morley plate bending element [33], which was then improved in [30] and was extended to the case of general boundary conditions [7]. A different approach [25] was taken in treating the case of a fully discontinuous interior penalty method [24], where the derivation of the reliability bound heavily depends on a suitable recovery operator mapping discontinuous finite element spaces into H_0^2-conforming spaces composed of high-order versions of the classical Hsieh-Clough-Tochner macro-element defined in [22]. The idea was also applied to establish an a posteriori bound for a quadratic C^0 interior penalty method for the biharmonic problem [8].

The aim of this paper is to construct reliable and efficient residual-based a posteriori error estimates of the moment-field error in an energy norm for the LCDG methods in [31]. Similar to the approach in [28], we make use of the Helmholtz decomposition in [6] to deduce the reliability (the upper bound). Particularly, two improved bounds are available when the orders of discrete spaces approximating the moment field and the displacement field satisfy some relation. As regards the efficiency (the lower bound), we follow the traditional lines [40] to bound all error indicators except the jump term with respect to the normal derivative of the approximating displacement field from above by the moment-field error in the energy norm plus the data oscillation.

The rest of the paper is organized as follows. In Section 2, we review the local C^0 discontinuous Galerkin methods for the Kirchhoff plate bending problem. An a posteriori error analysis is performed in Section 3. Finally, in Section 4 we report some numerical examples to illustrate the effectiveness of the error estimator.

We conclude the introduction with some basic notations used in the sequel. Given a bounded domain $\omega \subset \mathbb{R}^2$, we will use the usual L^2-based Sobolev space $H^s(\omega)$ $(s \geq 0)$ unless specified.