Journal of Computational Mathematics
Vol.29, No.6, 2011, 639–660.

http://www.global-sci.org/jcm
doi:10.4208/jcm.1109-m11si03

MULTIPLE TEMPERATURE GAS DYNAMIC EQUATIONS FOR NON-EQUILIBRIUM FLOWS*

Kun Xu
Mathematics Department, Hong Kong University of Science and Technology, Hong Kong, China
Email: makxu@ust.hk
Zhaoli Guo
National Laboratory of Coal Combustion, Huazhong University of Science and Technology, Wuhan 430074, China
Email: zlguo@mail.hust.edu.cn

Abstract

In an early approach, a kinetic model with multiple translational temperature [K. Xu, H. Liu and J. Jiang, Phys. Fluids **19**, 016101 (2007)] to simulate non-equilibrium flows was proposed. In this paper, instead of using three temperatures in the x, y and z-directions, we define the translational temperature as a second-order symmetric tensor. Under the new framework, the differences between the temperature tensor and the pressure tensor will be explicitly pointed out. Based on a multiple stage BGK-type collision model and the Chapman-Enskog expansion, the corresponding macroscopic gas dynamics equations in three-dimensional space will be derived. The zeroth-order expansion gives 10 moment closure equations similar to that of Levermore [C.D. Levermore, J. Stat. Phys **83**, pp.1021 (1996)]. The derived gas dynamic equations can be considered as a regularization of the 10 moments equations in the first-order expansion. The new gas dynamic equations have the same structure as the Navier-Stokes equations, but the stress-strain relationship in the Navier-Stokes equations is replaced by an algebraic equation with temperature differences. At the same time, the heat flux, which is absent in Levermore's 10 moment closure, is recovered. As a result, both the viscous and the heat conduction terms are unified under a single anisotropic temperature concept. In the continuum flow regime, the new gas dynamic equations automatically recover the standard Navier-Stokes equations. Our gas dynamic equations are natural extensions of the Navier-Stokes equations to the near continuum flow regime and can be used for microflow computations. Two examples, the force-driven Poiseuille flow and the Couette flow in the transition flow regime, are used to validate the model. Both analytical and numerical results are presented. Theoretically, the Boltzmann equation can be also applied to the current multiple stage gas evolution model to derive generalized macroscopic governing equations in the near continuum flow regime. Instead of using Maxwellian as an expansion point in the Chapman-Enskog method, the multiple temperature Gaussian can be used as an expansion point as well.

Mathematics subject classification: 65C20, 76P05.
Key words: Gas-kinetic Model, Multiple Translational Temperatures, Generalized Gas Dynamics Equations.

1. Introduction

The transport phenomena, i.e., mass, heat, and momentum transfer, in different flow regimes are of a great scientific and practical interest. The classification of various flow regimes is based

* Received January 1, 2011 / Revised version received April 21, 2011 / Accepted April 21, 2011 / Published online November 15, 2011 /

on the dimensionless parameter, i.e., the Knudsen number, which is a measure of the degree of rarefaction of the medium. The Knudsen number, Kn, is defined as the ratio of the mean free path to a characteristic length scale of the system. In the continuum flow regime where $Kn < 0.001$, the Navier-Stokes equations with linear relations between stress and strain and the Fourier's law for heat conduction are adequate to model fluid behavior. For flows in the continuum-transition regime ($0.1 < Kn < 1$), the Navier-Stokes (NS) equations are known to be inadequate. This regime is important in many practical engineering problems. Hence, accurate models that give reliable solutions at low computational costs, would be useful.

One of the alternative approaches to simulating non-equilibrium flow is that based on moment closures. Grad's 13 moment equations are among the most important. They provide the time evolution of non-equilibrium quantities, such as stress and heat flux [3]. However, due to its hyperbolic nature, these equations lead to a well-known sub-shock problem inside a shock layer when the Mach number is larger than a critical value. To improve the validity of the 13 moment equations, based on the Chapman-Enskog expansion, Struchtrup and Torrilhon introduced terms of the super-Burnett order to the balance of pressure deviator and the heat flux vector in the moment equations and established regularized 13 moment (R13) equations that have much better performance in the non-equilibrium flow regime [4]. Another well-known moment system is Levermore's 10 moment closure, which follows his hierarchy of non-perturbative moment closures with many desirable mathematical properties [2]. These equations do not suffer from closure-breakdown deficiencies, and they always give physically realizable solutions due to non-negative gas distribution functions. However, the 10 moment Gaussian closure has no heat flux even though the Navier-Stokes viscous terms can be recovered in the continuum flow regime. In an effort to extend the Gaussian closure to include higher-order effects, Groth et al. formulated perturbative variants of the original moment closure with a new extended fluid dynamics model [5]. The most studied of these closures is a 35-moment closure. Recently, McDonald and Groth took a Chapman-Enskog-type expansion of either the moment equations or the kinetic equation and introduced the heat flux into Levermore's 10 moment closures and obtained extended fluid dynamics equations for non-equilibrium flow simulation [6]. This new system leds to improved results in the transition flow regime where heat transfer has a significant effect. Other interesting research in this direction includes quasi-gas dynamic equations [7,8].

Currently, the Direct Simulation Monte Carlo (DSMC) method is the most successful technique for numerical prediction of low density flows [9]. The DSMC method primarily consists of two steps, i.e., free transport and collision within each computational cell. The determination of the transport coefficients in the DSMC method is based on the particle collision model, which is based on the well-defined theories for continuum flows. The collision models for the particle cross section and the probability for each collision pair can be used for recovering the dissipative coefficients in the Navier-Stokes limit. For example, the commonly used variable hard sphere (VHS) molecular model in DSMC can be used to recover the first-order Chapman-Enskog expansion with viscosity coefficient $\mu = \mu_{ref}(T/T_{ref})^{\omega}$, which is of the Navier-Stokes order. However, when the DSMC method is used for non-equilibrium flow calculations, the particle transport from one place to another is controlled individually by each particle's velocity, which is not uniformly controlled by the macroscopically defined particle collision time τ, i.e., $\tau = \mu/p$, where μ is the dynamic viscosity coefficient and p is the pressure. Therefore, particle transport from one place to another in DSMC may be the key for capturing of non-equilibrium properties. Traditionally, it is noted that the concepts and measurements of the dissipative coefficients are limited to the continuum flow regime. A generalized mathematical formulation